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Rotating convection at moderate Prandtl number

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Abstract

A small rotation is shown to inhibit the spiral chaos which develops in Rayleigh–Bénard convection at moderate Prandtl number. Near the critical rotation for the onset of the Küppers–Lortz instability, large coherent targets embedded in a turbulence background are formed in the case of free-slip top and bottom boundary conditions, while in the case of no-slip boundary conditions, unfolding of the spirals is observed and, for periodic horizontal geometry, straight parallel rolls can reform. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

Rotating convection is often used as a prototype for transition to spatio-temporal chaos [1], due to the destabilization of parallel-rolls near the onset by the Küppers–Lortz (KL) instability when the rotation rate τ, defined as twice the Rossby number, exceeds a critical value τ_{KL} [2–5]. At infinite Prandtl number, $\tau_{KL} = 47.8$ in the case of free-slip top and bottom boundary conditions and $\tau_{KL} = 54.8$ for no-slip boundaries. The nonlinear dynamics then results from the formation of patches of parallel rolls which penetrate each other in a chaotic way, with directions tilting by an angle close to 60° [6].

It turns out that a much richer dynamics develops at moderate Prandtl number, due to the presence of a mean flow whose amplitude is especially strong in the case of free-slip boundary conditions, a configuration often relevant in astrophysical and geophysical contexts. The aim of the present paper is to briefly discuss some recent results concerning rotating convection at finite Prandtl number, with special insight on the competing effects of the mean flow and of a small rotation. In some instances, a moderate rotation leads to the unfolding of the spirals obtained in the absence of

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rotation or to the formation of large-scale coherent structures, according to the prescribed boundary conditions.

2. Linear and weakly nonlinear dynamics

In a rotating convecting fluid at finite Prandtl number with free-slip top and bottom boundary conditions, straight rolls are unstable relatively to quasi-parallel perturbations. This "small-angle instability" occurs for arbitrary slow rotation and its growth rate scales like $\varepsilon^{4/3}$ with the normalized distance from threshold ε, thus dominating that of the KL instability which varies like ε^2. A derivation is presented in Ref. [7], on the basis of a matched asymptotic-expansion performed on the Boussinesq equations. In this context, the usual computation of the KL instability which displays a divergence for quasi-parallel perturbations [5], appears as an outer expansion restricted to perturbation angles outside an angular boundary layer whose thickness scales like $\varepsilon^{2/3}$. Resulting from a vertically uniform contribution of the mean flow, this new instability does not survive with rigid boundary conditions.

In order to analyze its nonlinear development and more generally to study collective aspects of rotating convection with free-slip or rigid boundary conditions, a generalized Swift–Hohenberg (SH) model for the convective mode W and the mean-flow stream function Ψ was developed in Ref. [8] in the form

$$\tau_0 \partial_t W = (\varepsilon - (\Delta + 1)^2) W - \mathcal{N}(W, \Psi)\,, \tag{1}$$

$$(\partial_t - P(\nu + \Delta))\Delta\Psi = (\nabla\Delta W \times \nabla W)\cdot\hat{\mathbf{z}} + \lambda[(\Delta W)^2 + \nabla W \cdot \nabla\Delta W] + \mu\Delta(W^2)\,, \tag{2}$$

which can be viewed as an extension to the rotating convection of a system derived in Ref. [9]. Eq. (2) for the mean-flow results from a systematic asymptotic expansion. It includes for rigid boundaries, a friction coefficient $\nu = \pi^2/q_c^2$ where the value of the critical wave number q_c increases with the rotation rate. The numerical coefficients τ_0, λ and μ depend on the Prandtl number, the rotation rate and the nature of the boundary conditions. The parameter μ vanishes with free-slip boundary conditions, the term it multiplies originating from the vertical Reynolds stress. In Eq. (1), the nonlinear term $\mathcal{N}(W, \Psi)$ is obtained by simplifying the result of an asymptotic expansion in a way which accurately preserves the KL and (for free-slip boundary conditions) the small-angle instabilities, together with the onset of the zig-zag instability at zero rotation.

For free-slip boundaries, integration of the SH model, Eqs. (1)–(2), with periodic boundary conditions very near threshold shows that the mean flow develops shear layers and that the distortion of the rolls and their reconnection leads to a global rotation of the pattern in the direction of the external rotation, as suggested by the sign of the vertical vorticity generated by the term including the factor $\lambda = -\pi^2\tau/[q_c^2(q_c^2 + \pi^2)]$ in the mean-flow equation.

In the weakly nonlinear regime, it is also of interest to analyze the effect of a phase modulation on straight parallel rolls. In the absence of rotation, these structures are known to be unstable to a skewed-varicose instability when the wave number is larger than critical [10], a phenomenon recovered by computing the Busse stability balloon in the context of the present model. Denoting the normal modes of the phase perturbation by $e^{i(\kappa\cdot x+\sigma t)/\eta}$, where η^{-1} is the aspect ratio and $\kappa = (|\kappa|\cos\rho, |\kappa|\sin\rho)$, we obtain, as usual, a growth rate which is symmetric in terms of ρ. In contrast, when a rotation is turned on, straight rolls are always unstable and the skewed-varicose growth rate becomes asymmetric. When $k > q_c$, it is maximum for a value of ρ whose sign is that of the external rotation, while when $k < q_c$, it is maximum for an angle of opposite sign. In physical space, the above perturbation leads to regions of compression and dilatation of the rolls, alternating along an axis making the same angle ρ with the basic wave vector, while the associated mean flow displays shear layers perpendicular to this axis. Numerical integration of the SH model shows that, according to the sign of the angle ρ, the perturbation is amplified or not, as predicted by the modulation analysis.

The small angle and the asymmetric skewed-varicose instabilities thus lead to a similar pattern dynamics in physical space. Furthermore, it is easily seen that the two growth rates scale like the inverse aspect ratio η. The nature of the perturbations nevertheless, are, different. For the small-angle instability, the perturbation eigenmode consists in a single Fourier mode which is unstable if the angle of its wave vector with that of the basic rolls has the sign of the external rotation. In the phase formalism, in contrast, it can be viewed as a pair of satellites whose separation scales like η. In a large box, the interaction of satellites is resonant and a mode which alone would be unstable, is stabilized by the presence of its companion, as predicted by the phase theory. When the wave number is distinct from critical, the sign of the resulting rotation of the pattern can be predicted by noticing that among the two satellites associated with the phase perturbation, the closest to the critical circle will be preferentially amplified and when after a while it becomes dominant, the pattern undergoes a dynamics prescribed by the small-angle instability, possibly leading to the reversal of the rotation direction of the pattern. We conclude that the small-angle and the asymmetric skewed-varicose instabilities can be viewed as two representations of the same physical process.

3. Fully nonlinear dynamics

For larger values of the stress parameter ε and in the absence of rotation, the parallel rolls are replaced by the now well-documented spiral turbulence. The effect of weak-rotation depends on the boundary conditions [11]. With free-slip top and bottom boundaries and horizontal periodicity, targets of moderate size, associated to patches of vorticity of the same sign as that of the external rotation are formed and they subsequently grow by accretion of adjacent rolls and by merging together, leading to a big coherent structure occupying the whole domain (Fig. 1). In the case of a

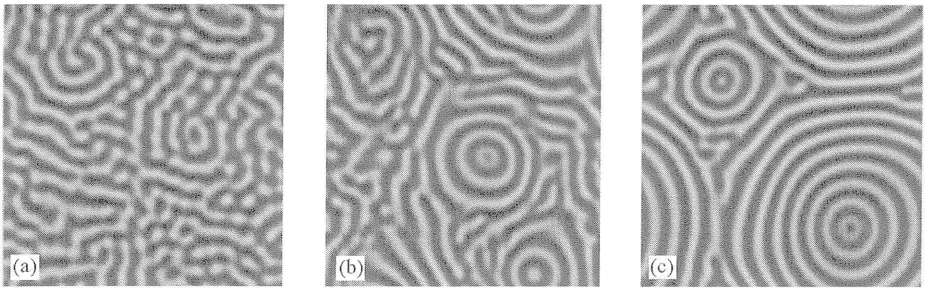

Fig. 1. Formation of large-scale targets in rotating convection with free-slip boundaries for $P = 2$, $\tau = 10$ and $\varepsilon = 0.7$.

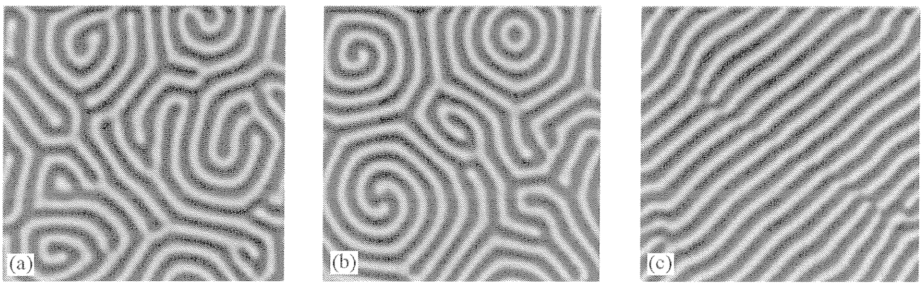

Fig. 2. Typical snapshots for rigid boundary conditions at $P = 1.2$, $\varepsilon = 0.7$ and different rotation rates $\tau = 0$ (a), $\tau = 10$ (b), $\tau = 40$ (c) (to be compared to $\tau_{KL} = 29.6$).

cylindrical box, the pattern adjusts to the symmetry of the container, leading to perfectly concentric rolls filling the whole box. At higher rotation, the coherent structures are destroyed by the KL instability and a fully chaotic regime develops.

In the case of rigid top and bottom boundary conditions, the effect of a small rotation in periodic geometry at moderate Prandtl number, is to increase the size of the spirals and targets and to force their rotation in the direction of the external rotation as observed in laboratory experiments [12], vorticity patches with corresponding sign being formed at their center. As the rotation rate is increased, the spirals grow in size and their number is reduced. When it slightly exceeds the critical value τ_{KL} for the onset of the KL instability, the pattern evolves to almost straight parallel rolls, swept by gliding dislocations which gradually annihilate by collision (Fig. 2). When the rotation rate is further increased, a fully chaotic dynamics again develops.

In cylindrical geometry, the dislocations cannot annihilate each other as efficiently as in a periodic domain. As a result, although a significant unfolding of the spirals is observed for intermediate values of the rotation rate, the pattern does not reduce to straight rolls. A noticeable feature is also that the pattern globally rotates (even when the rotation rate is smaller than τ_{KL}), under the effect of dislocations generated on the sidewall, an effect also noted in laboratory experiments [13].

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