Inferring the sub-surface rotational gradients using f-modes and ridge fitting analysis

T. Corbard

Observatoire de la Côte d'Azur, France

J. Reiter

Technische Universität München, Germany

M.J. Thompson

University of Sheffield, U.K.

Abstract

In a previous work *(Corbard & Thompson, 2002)*, MDI f-modes with spherical harmonic degrees up to ℓ=300 have been used to infer properties of the radial gradient of angular velocity in the near sub-photospheric layers. This information is very important in order to better constrain numerical models of the convection in this layer.

Local helioseismology, however, gave hints that the resolution reached by our global mode analysis might not be enough to reveal the full complexity of the angular velocity gradients and their radial variations.

In this work we use frequency splittings estimated from a ridge fitting technique for degrees up to ℓ=1000 *(Reiter, 2007)* in order to reach more resolution in the radial direction close to the photosphere. We show that the linear assumption used in the previous work cannot be kept anymore when using this new dataset and present our first results obtained by inverting the new data.

Motivations (1)

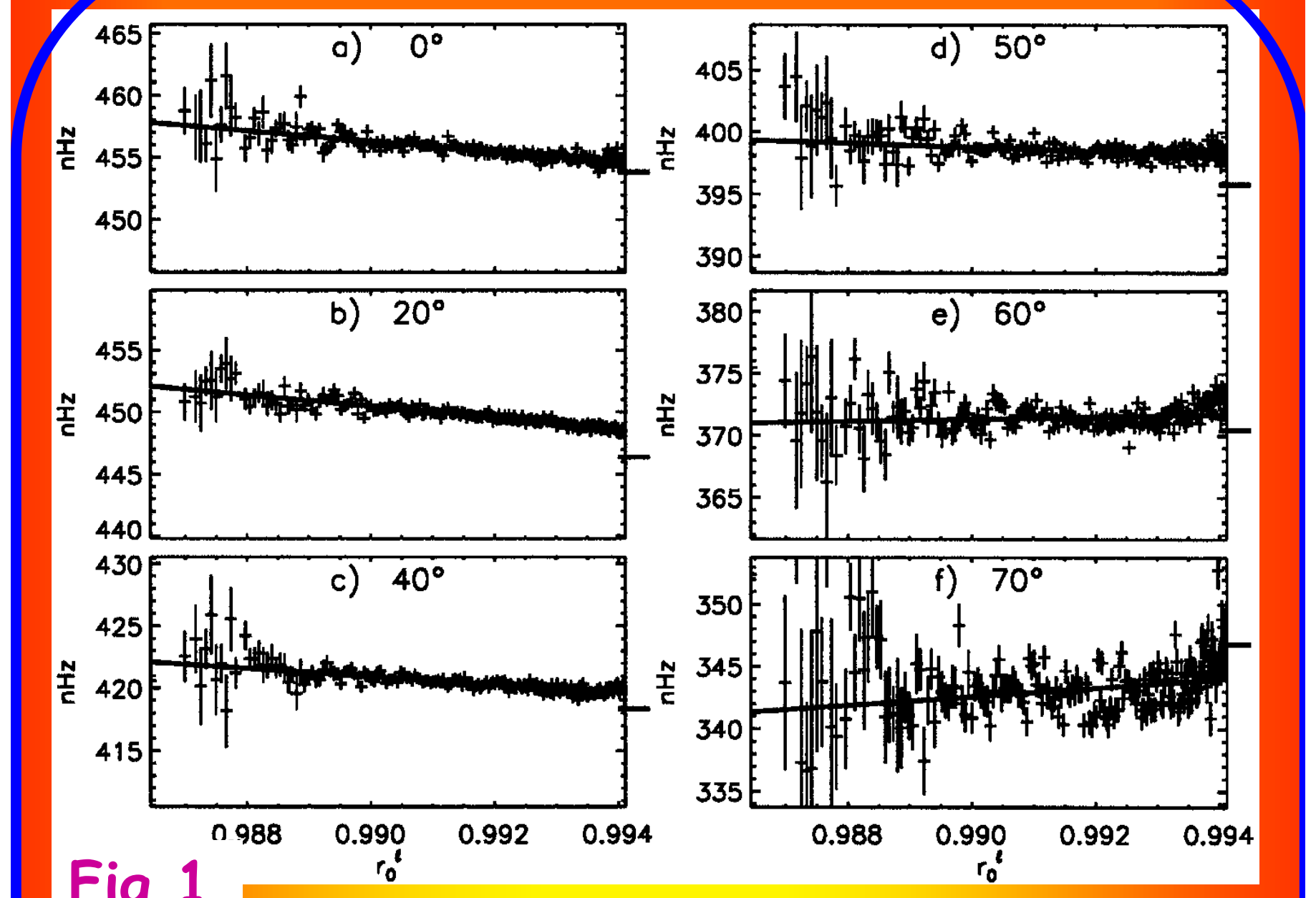

Fig 1

Fig. 1 shows the result obtained by *Corbard & Thompson (2002)* using MDI f-modes splittings. These plots show the rotation rate as a function of depth in near surface layers and for different latitudes. Using the fact that f-modes rotational kernels have only one lobe, we didn't use inversion but instead just plotted the value obtained for a given mode at the location of the center of gravity of its kernel. At the shallowest depths and high latitudes the linear approximation, however, seems less justified.

Motivations (2)

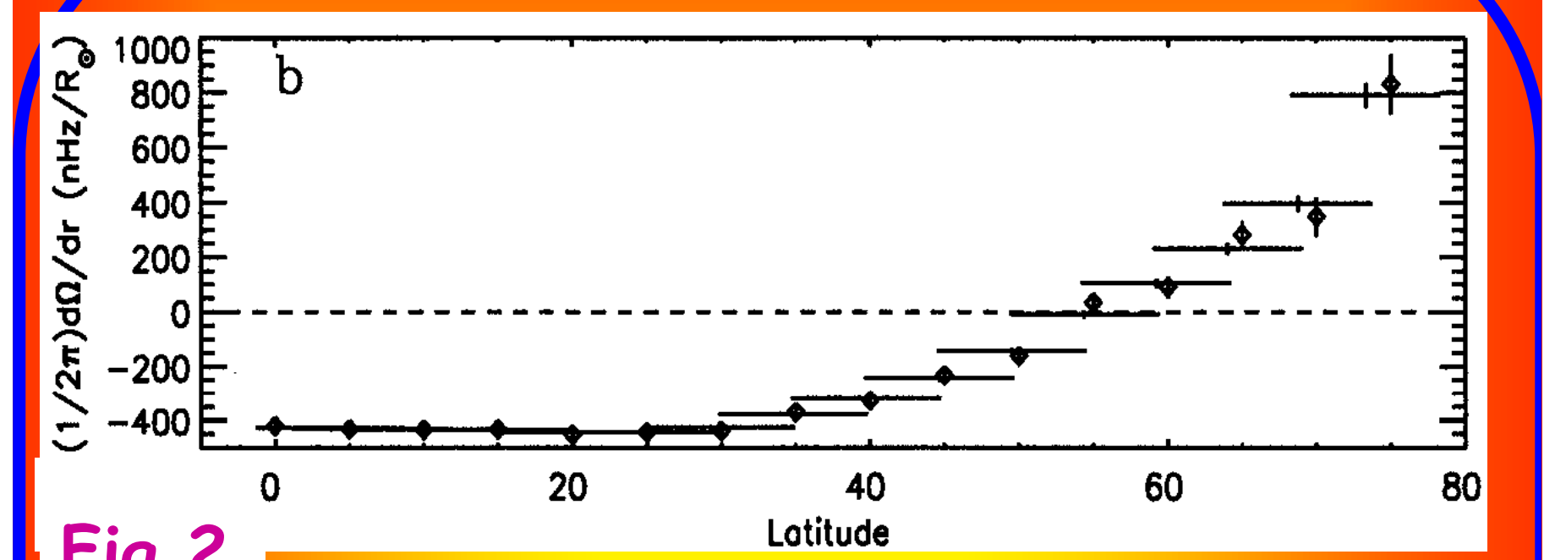

Fig 2

One important result reached in our previous study concerned the value of the linear angular velocity gradient found at low latitudes. Fig. 2 is a plot of the slopes of the linear fits plotted in Fig. 1 for each latitude. It shows a negative gradient of about -400 nHz/R for all latitudes below 40°. This corresponds to a logarithmic derivative of -1 whereas one would expect -2 if moving parcels of fluid were to conserve their specific angular momentum as they move towards or away from the rotation axis. This is, however, in agreement with the latest numerical simulations of rotating compressible convective fluid *(De Rosa et al., 2002)*. This quantity therefore represents an important diagnostic for our understanding of the physical properties of the sub-surface layers and the goal of this work is to extend our analysis to shallowest layers by analyzing very high degree modes (300<ℓ<1000) as obtained from a ridge fitting technique.

Ridge Fitting Technique (1)

For the ridge fitting we employ the **multiple-peak, tesseral-spectrum (MPTS) method** *(Reiter et al. 2002, 2003; Reiter 2007).*

This MPTS method operates directly upon of all the modes in a multiplet of given (n, ℓ), and it employs a sum of a series of several different individual peaks, each of which can be either a symmetric Lorentzian profile or the asymmetric profile of *Nigam & Kosovichev (1998).*

As a result, 2ℓ+1 sets of modal parameters (amplitude, frequency, line width, line asymmetry, background) are obtained for each multiplet (n, ℓ). By expanding the 2ℓ+1 estimated frequencies $\nu_{n\ell m}$ in a multiplet in either the orthogonal polynomials introduced by *Ritzwoller & Lavely (1991)* or in Legendre polynomials the mean-multiplet frequency $\nu_{n,\ell}$ and the a-coefficients of that multiplet can be derived.

Because the power in a single frequency bin of a tesseral, zonal, or sectoral spectrum obeys an exponential rather than a Gaussian distribution the mode parameters have to be estimated by a maximum likelihood method rather than by an ordinary least-squares approach.

The results presented here were obtained by using the asymmetric profile of *Nigam & Kosovichev (1998)* and the *Ritzwoller & Lavely (1991)* polynomials throughout.

Comparison with "standard MDI splittings"

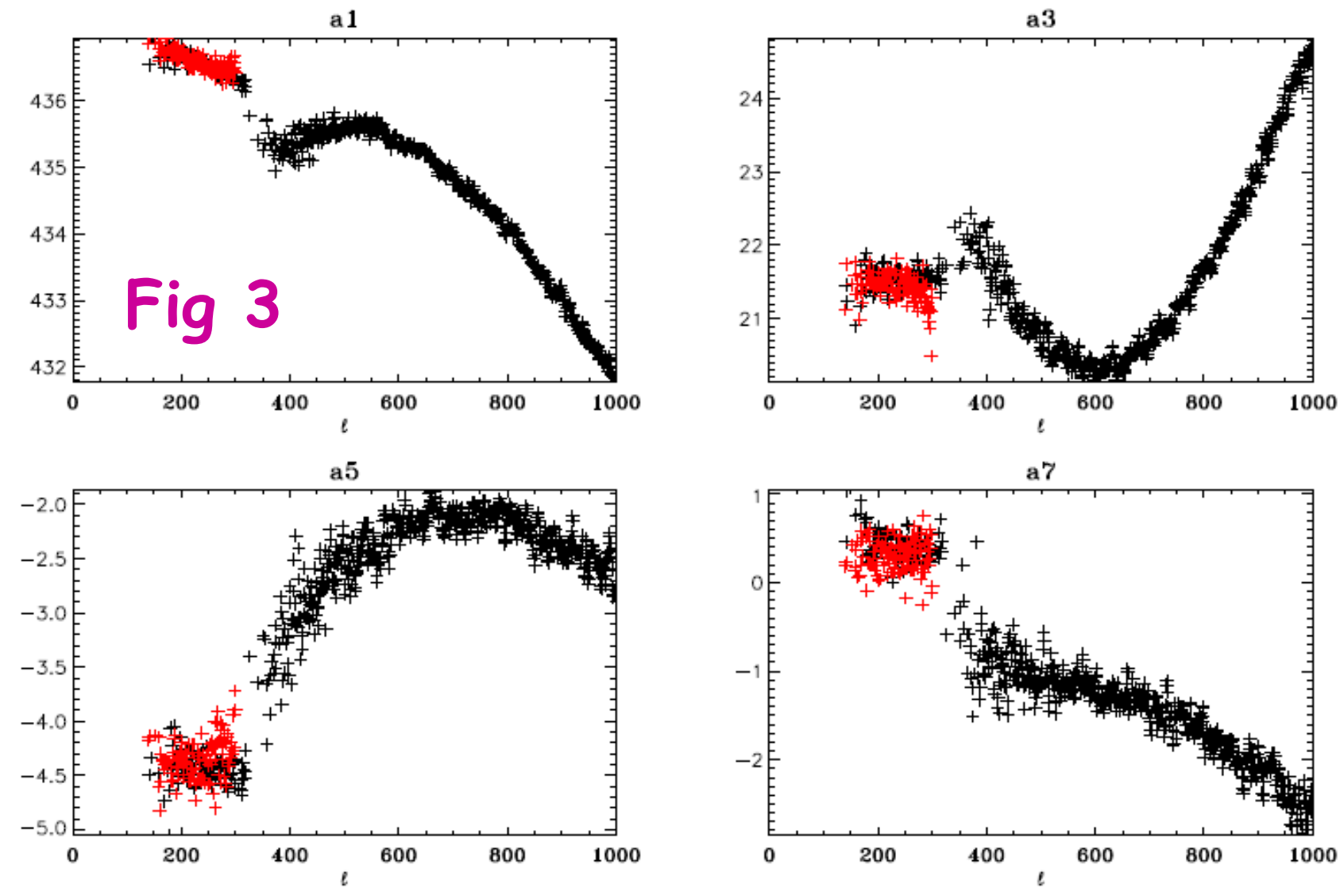

Fig 3

Our data (in black) covering the period 23 May 23 – 24 July 1996 are compared with J. Schou "standard" splitting coefficients (in red) covering the closest period (1 May-12 July 1996) *(Schou 1999)* . We have shifted the "standard" a_1 down by 31.0 nHz to account for the sideral-synodic conversion. It looks quite compatible except near ℓ=300 which corresponds to the edge of J. Schou fitting range.
For our analysis we used only modes for which we have fitted at least up to a_7. This results in a sparse region between ℓ=320 and 350. The gap is happening where a_7 goes from positive to negative and is the result of the fitting criterion that reject small values.

The linear assumption and its limitations

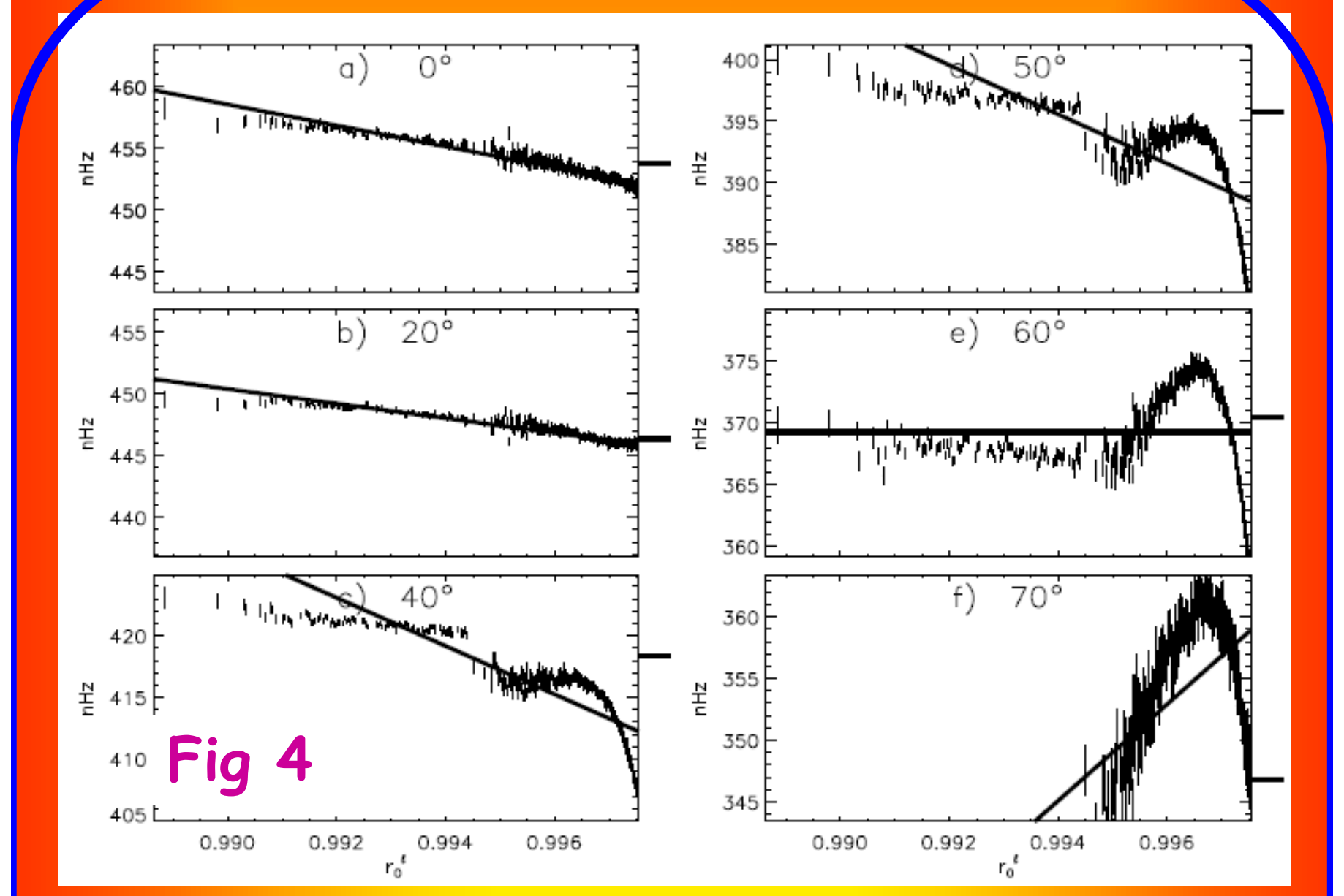

Fig 4

This the same as Fig. 1 but using our new data. The higher ℓ values allow us to extend the radius range up to 0.9975R .The data gap at ℓ=320 corresponds to a change of regime for latitude above 40°. The dramatic behavior in the rotation rate comes from the large trend in the high ℓ splittings. If real this would imply extremely large gradient of rotation rate in the immediate sub surface.

At low latitudes, the coefficients cancel out and the rotation variation keeps roughly linear. The slope found at low latitudes from a simple linear fit leads to a logarithmic derivative value close to -2 in apparent contradiction with our previous results. However, even at these latitudes the linear assumption appear less valid and must be questioned.

The linear assumption and its limitations (2)

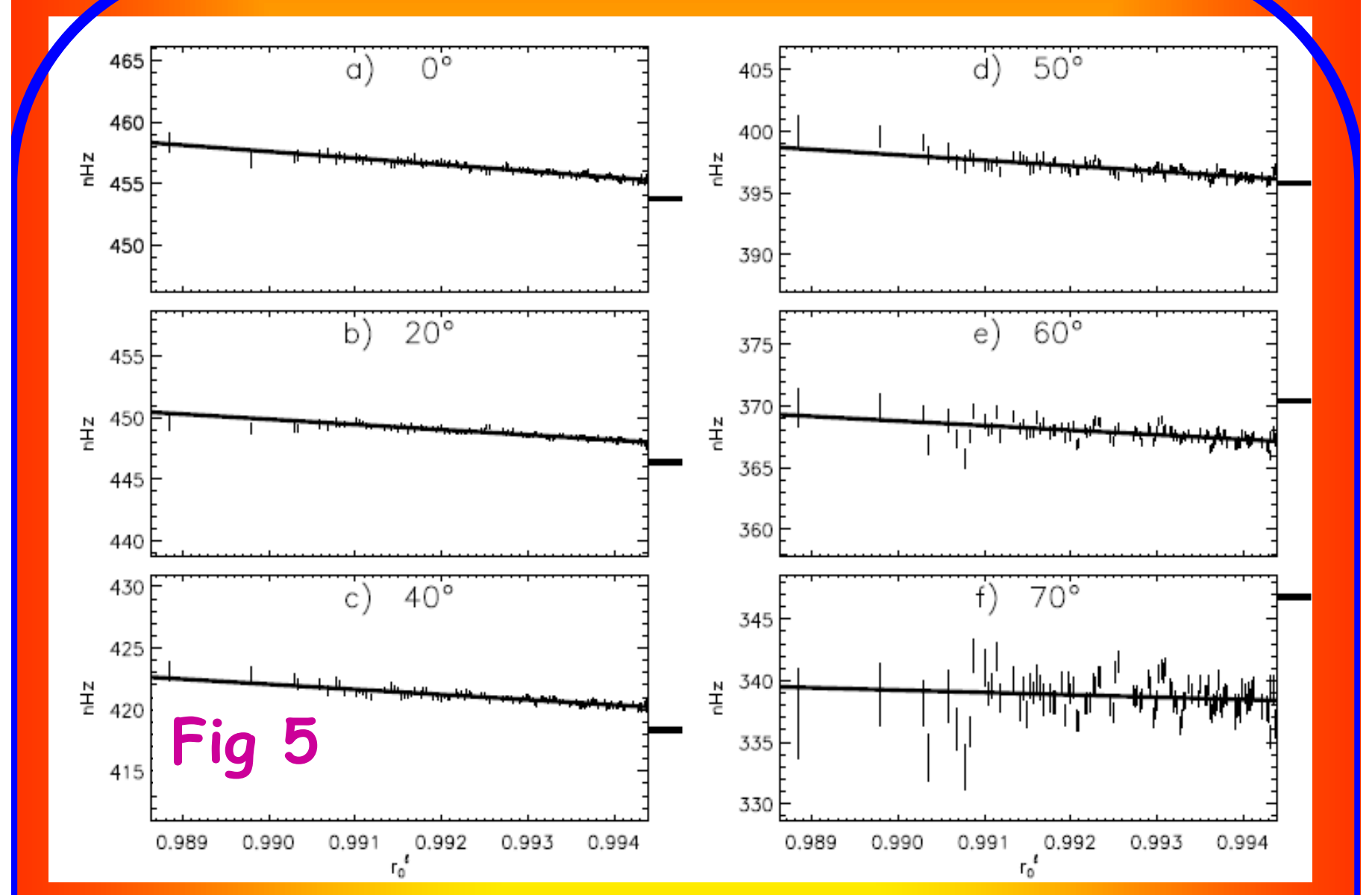

Fig 5

As a further experiment we have restricted the analysis to ℓ<320 and these results are shown here. The slope and linear trends are very well recovered. The behavior at the edge (around 0.994R) seems even more linear than it was in our previous work at 60°.
The results at 70°are not to be trusted because we can't localize kernel that high with only 4 a-coefficients.
It is interesting to notice that the slope at 60°is still negative with the new dataset while it was changing sign in Fig. 1. This confirms that the sign reversal at this latitude is probably confined to the shallowest layers.

About latitudinal resolution

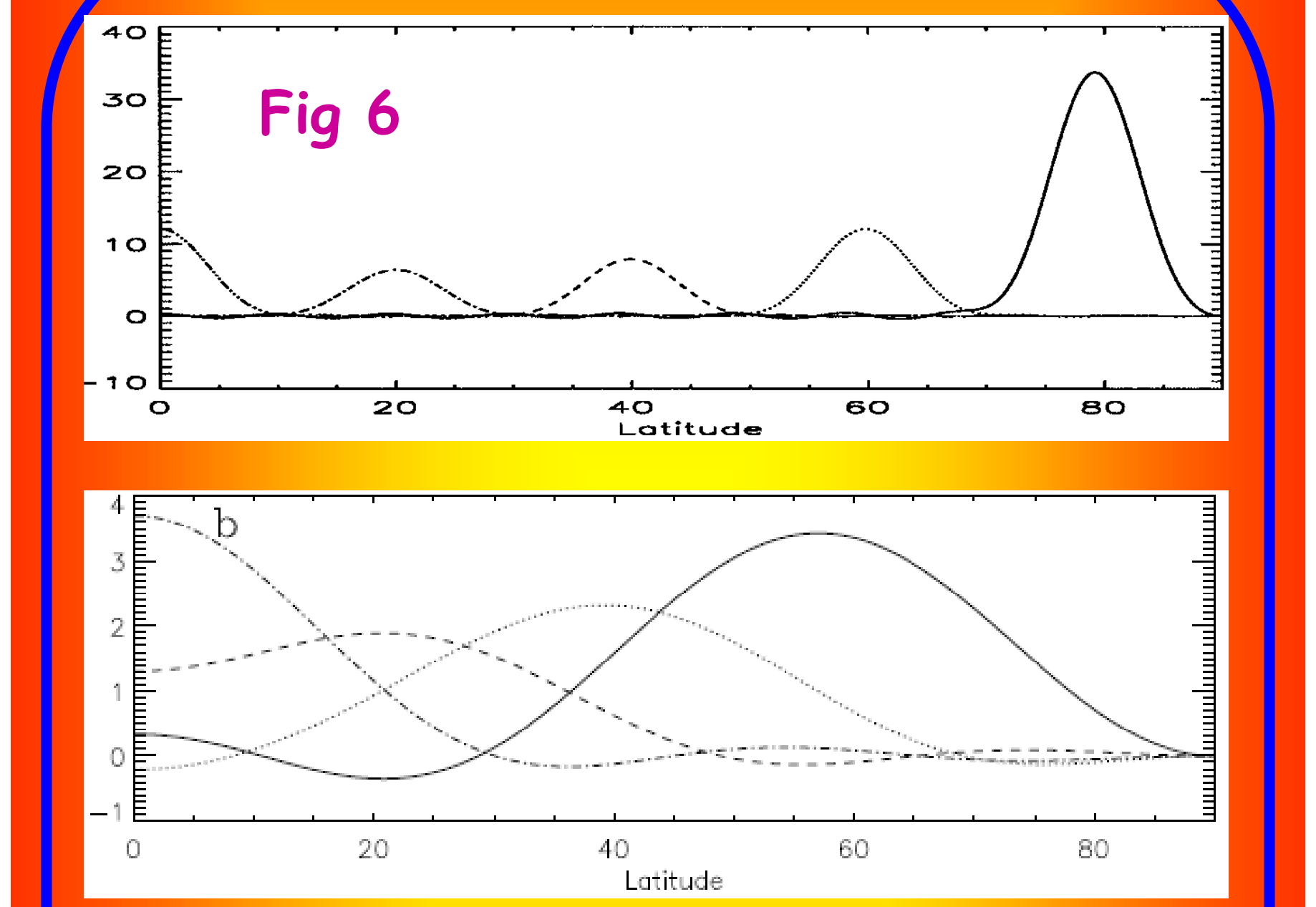

Fig 6

Latitudinal averaging kernels showing the latitudinal resolution of the method for a given dataset. With the 18 odd coefficients of the standard MDI dataset we could reach about 10 degree latitudinal resolution as shown on the upper figure. The corresponding figure for the new data with 4 a-coefficients only is given below. The resolution reached is about 20 degrees and we can not localize a kernel above 60 degree of latitude.

OLA inversion of the rotation rate

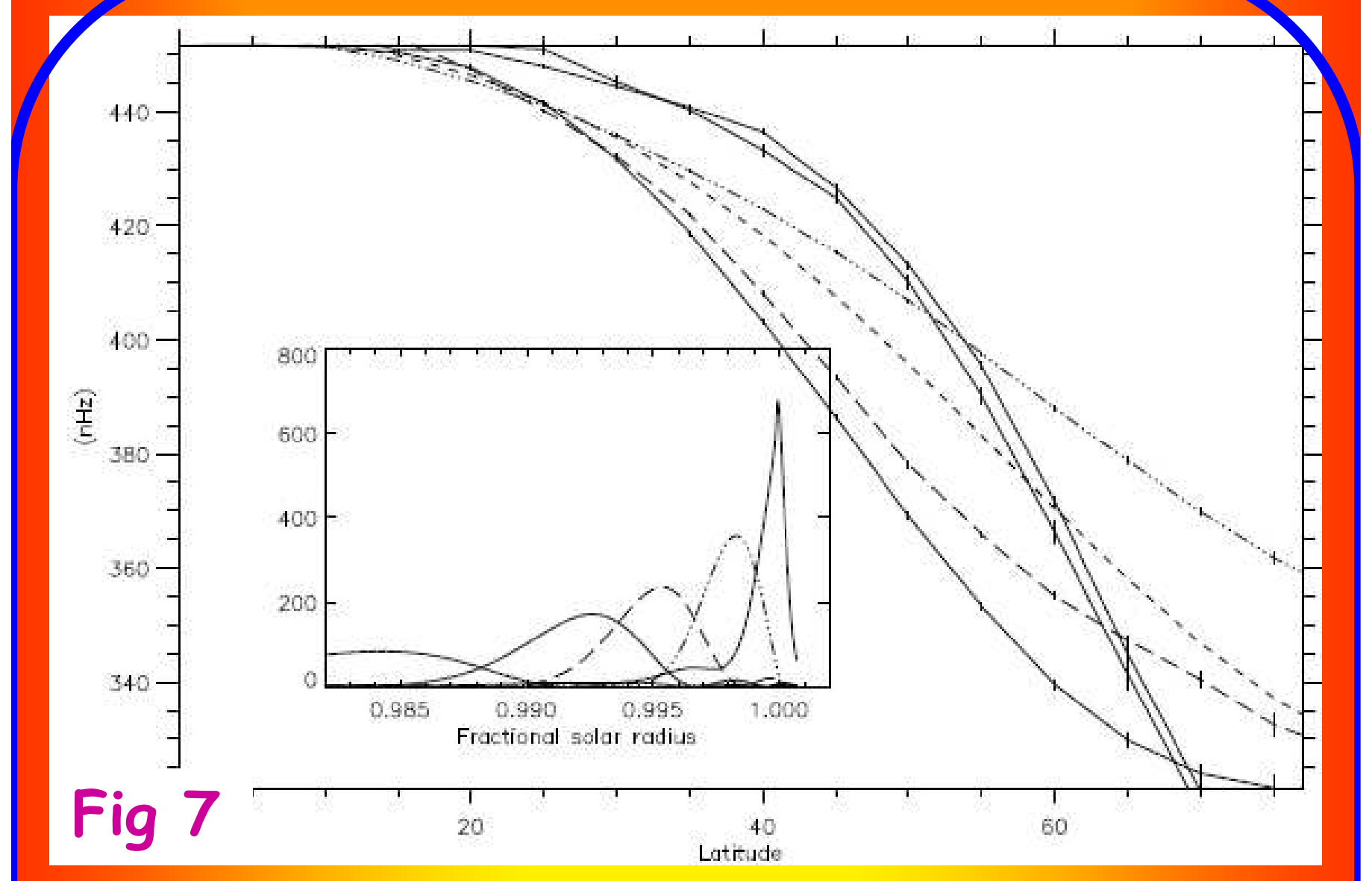

Fig 7

From Fig. 4 we realized that the rotation rate doesn't vary linearly with depth in the whole domain reachable with the new data and therefore such a representation is not appropriate anymore. Therefore we attempt to do radial inversion in order to localize averaging kernels at several depths to see how the rotation varies.
You can see here the result as a function of latitude for 5 depths materialized by the location of the kernels in the sub-panel. (The additional dashed line being the Snodgrass surface rotation rate.)

OLA inversion of the rotational gradient (1: test on the standard data)

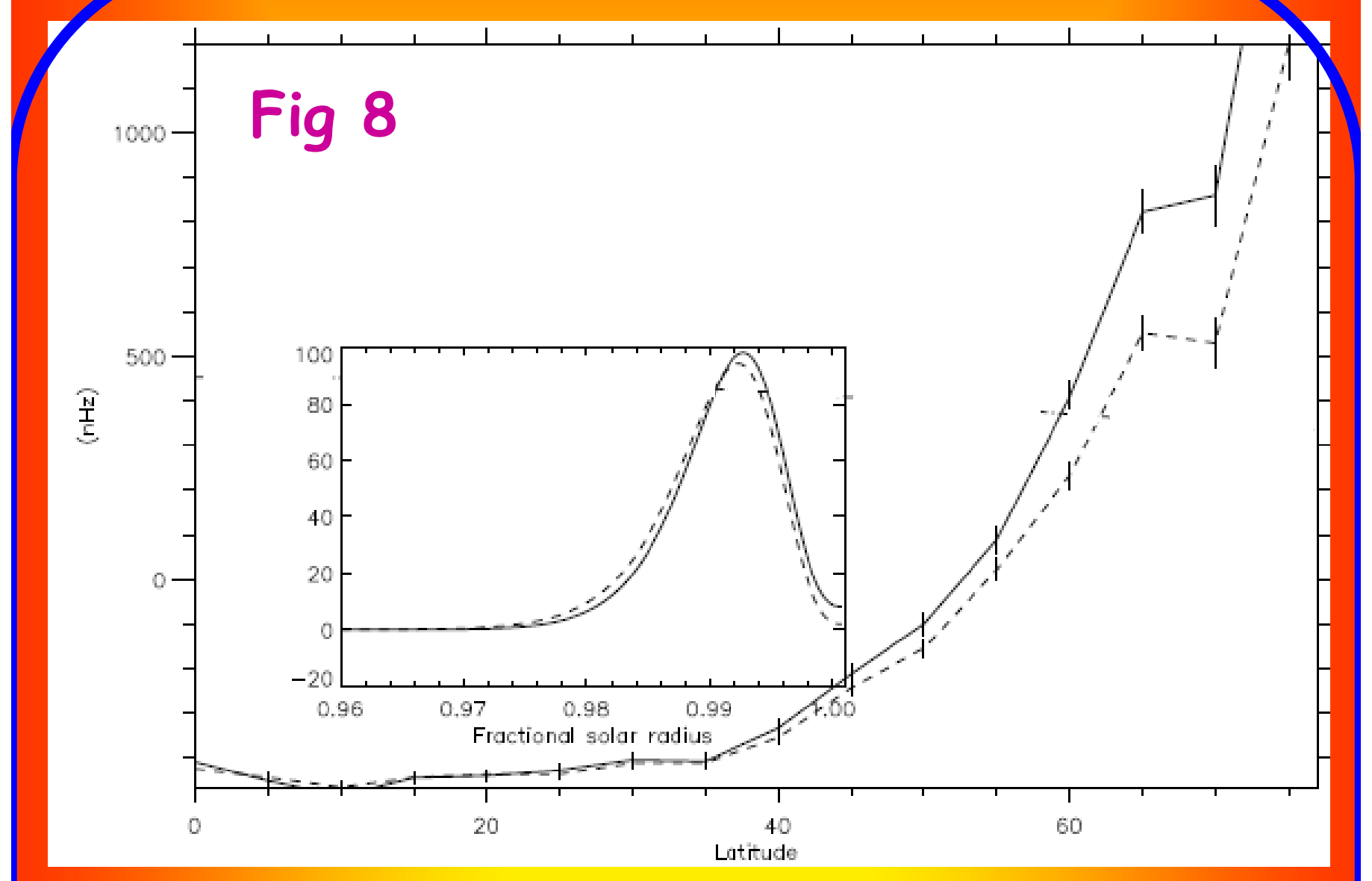

Fig 8

Since our main dynamical interest is in getting the rotational gradient we thought to estimate this radial gradient from the rotation rate curves obtained at different depths (Fig.7) in order to find back a figure similar to Fig.2 for the new dataset. However a numerical derivative on such a small radial range is dominated by errors. We therefore thought to make inversion not for the rotation rate but directly for its derivative *(Thompson, 1990)* . We have first tested this approach by doing that on the standard MDI data and we have obtained the curves above which match quite well the result of Fig.2

OLA inversion of the rotational gradient (2:Results using the new data)

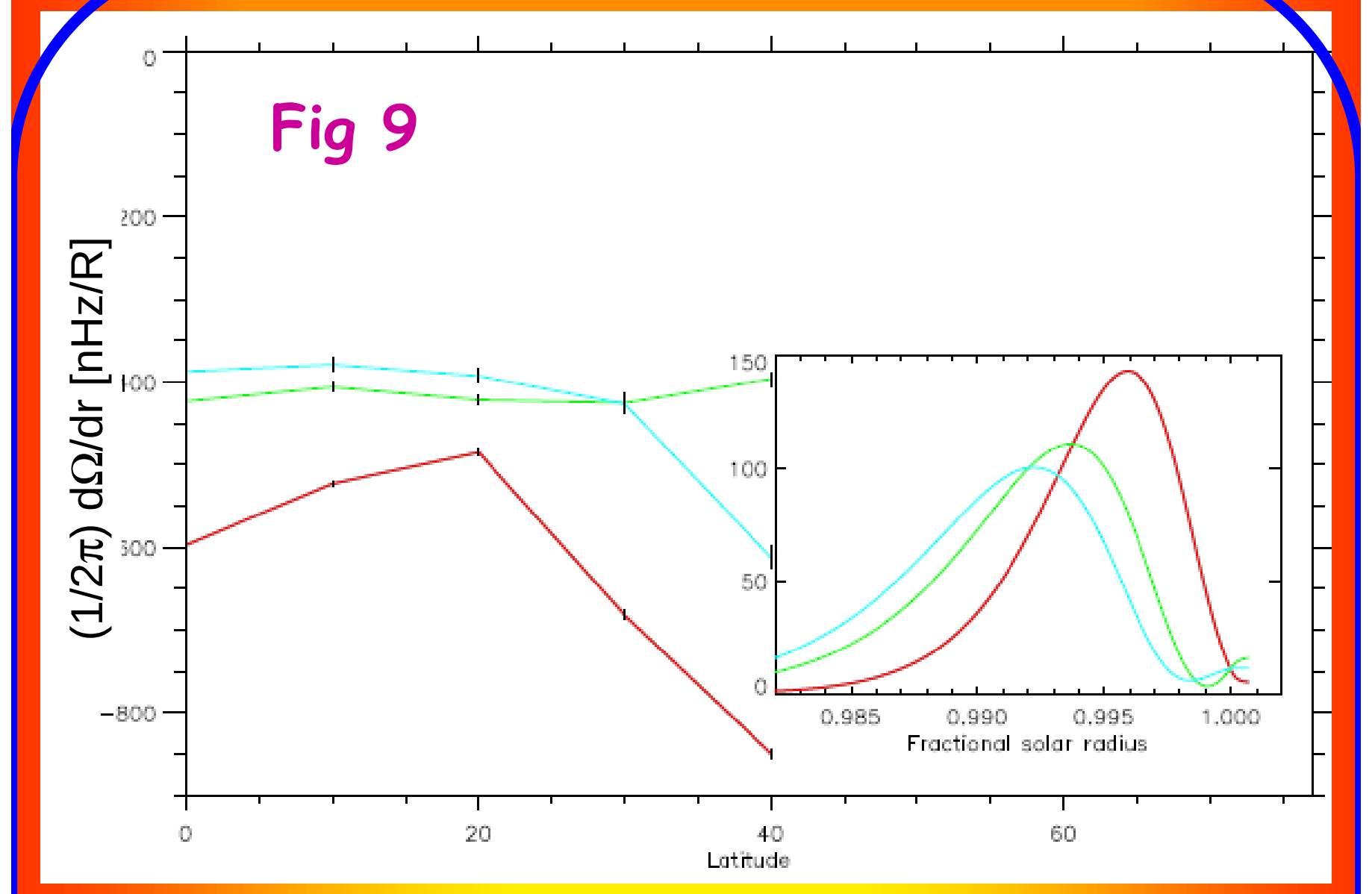

Fig 9

You can see here the result reached using the new data; we estimated the rotation rate gradient at 3 different depths restricting ourselves to latitudes lower than 40 degrees. It basically shows that the gradient get steeper towards the surface, the value of -400 nHz/R reached for the deeper point being equivalent to the value reached in our previous work for those latitudes.
We don't quite believe the results obtained when we try to go at higher latitudes or closer to the surface. As a matter of fact, as you can notice in Fig. 7 the curve for the shallowest rotation profile (the isolated full line) is pretty far from the Snodgrass curve (dashed)

Conclusions & Perspectives

- ☐ The ridge fitting method developed produce splitting coefficients in good agreement with MDI standard splittings within the range of spherical harmonic they share. This preliminary work is encouraging concerning our ability to obtain useful information about the near sub-surface layers using frequency splittings of very high degree modes deduced from the ridge fitting.
- ☐The first results indicate that the linear assumption for the radial variation of the rotation rate is not valid anymore when the shallowest layers are considered.
- ☐By using direct inversion of the rotational gradient, we were able to confirm our previous results concerning the amplitude of this gradient at low latitudes. This preliminary analysis of the splittings of the high degree modes indicates, however, that the angular velocity gradient increases as we go closer to the photosphere.
- ☐The limited amount of a-coefficients available for high degree modes do not allow us to conclude about the sign reversal of the angular velocity gradient found around 55° in our previous work. However, like in our previous work, we notice that excluding the highest degree modes result in canceling the sign reversal which indicate that this reversal is probably confined in the shallowest layers.
- ☐In order to complete this work, we plan to analyze data sets covering a wider period and we will attempt to use individual splittings instead of a-coefficient in order to reach higher latitudes.

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Observatoire de la Côte d'Azur

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