Low dimensional turbulence

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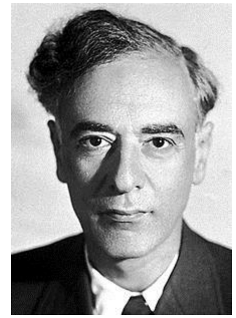
Turbulence: 2 non reconcilable viewpoints

- **Landau (1944):**



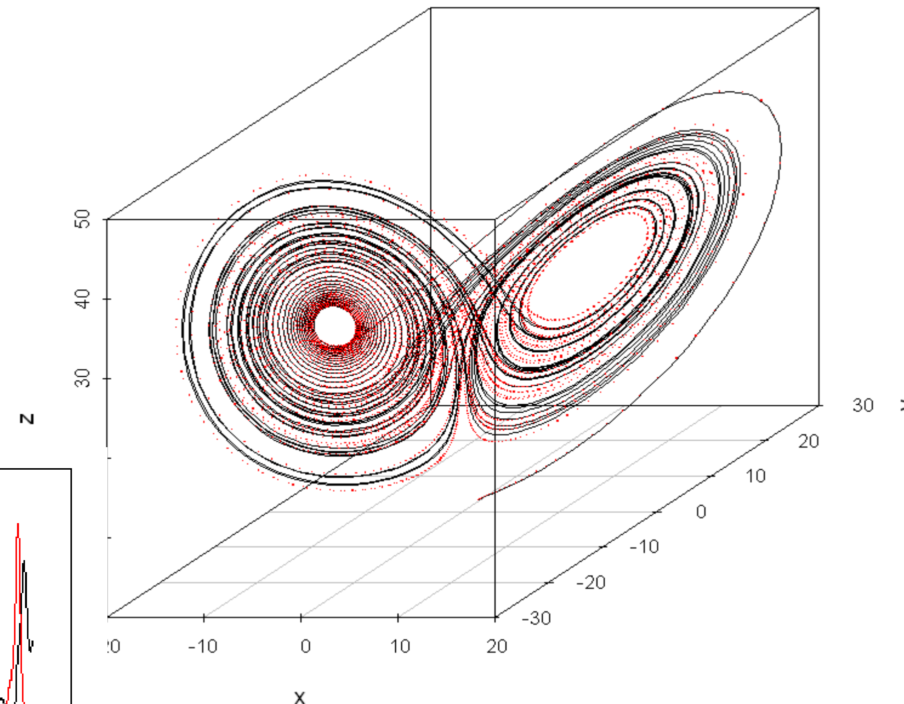
Turbulence = superposition of a growing number of modes with incommensurate oscillation frequencies, resulting from an infinite number of bifurcations with increasing Re

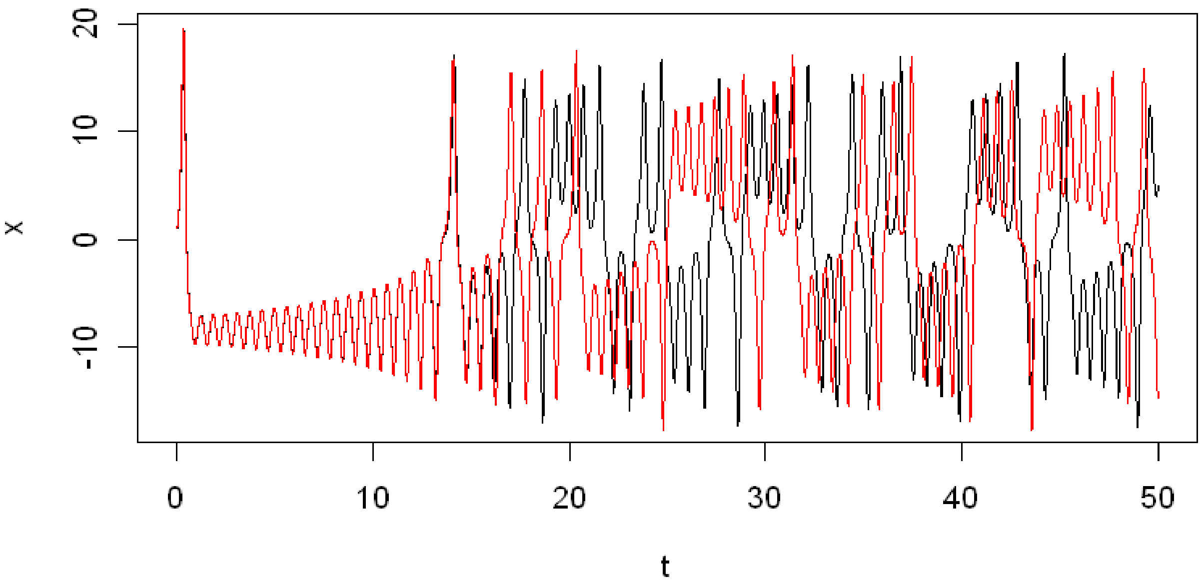
- **Ruelle and Takens (1971):**

Turbulent states = described by a small number of degrees of freedom, i.e. by a low dimensional "strange attractor" on which all turbulent motions concentrates in a suitable phase space

Lorenz attractor (63)

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = \rho x - y - xz, \quad \sigma, \beta, \rho \geq 0, \\ \dot{z} = -\beta z + xy. \end{cases}$$





Turbulence and symmetry breaking (1)

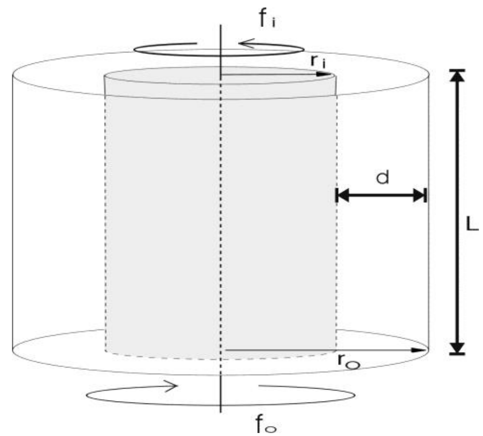
- Transition to turbulence: succession of symmetry breakings/bifurcations of the flow

→ 2 main types of transitions:

- supercritical, continuous
  Ex: Rayleigh Bénard convection or Taylor Couette flow

- subcritical, discontinuous, finite amplitude solutions
  Ex: plane Couette or Taylor Couette flow

Transition to turbulence in Taylor Couette



laminar

stationary pattern

oscillating pattern

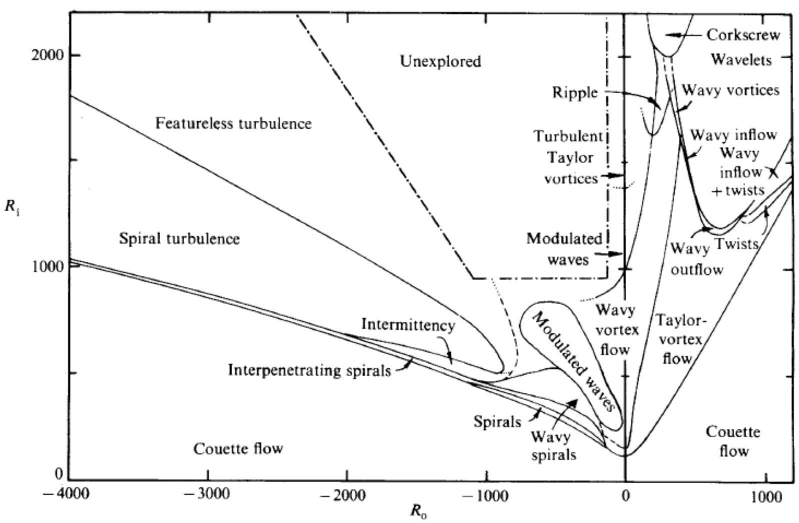
chaos

turbulence

……

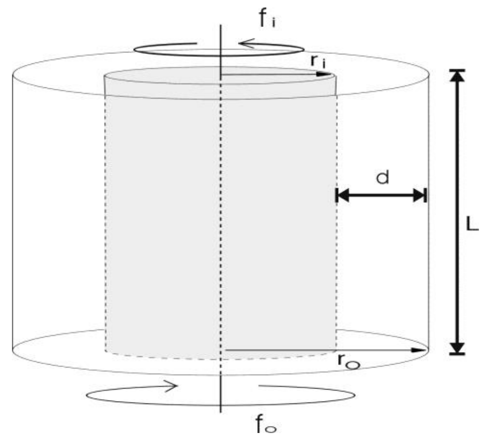
End of story?

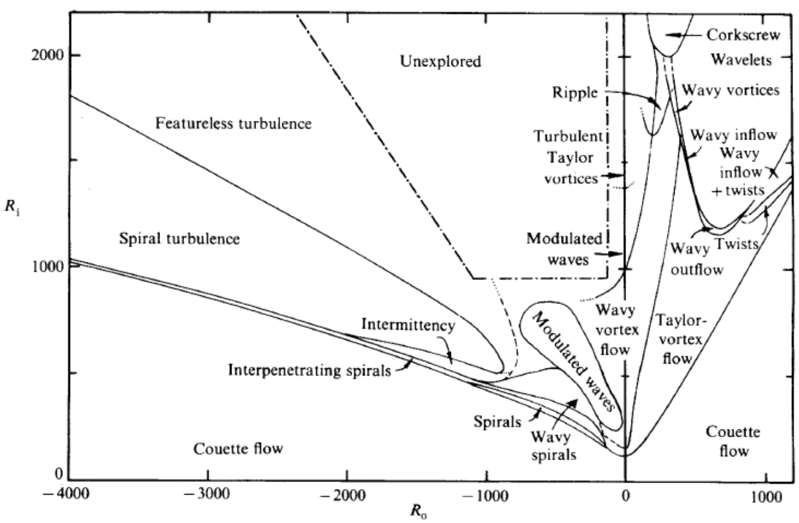
Re



Andereck et al. JFM 86

Transition to turbulence in Taylor Couette





Andereck et al. JFM 86

?

Laminar/mean state

stationary pattern

oscillating pattern

chaos

turbulence

……

End of story?

Re

Turbulence and symmetry breaking (2)

- at “large” Reynolds number *Re:*
  → Turbulence is fully developed (K41, intermittency..)
  → Symmetries are statistically restored *(Frisch 95)*

- However, at large *Re*, observation of new symmetry breakings which concern the flow statistics :

Turbulent bifurcations, instabilities…chaos?

→ in natural systems

→ in laboratory experiments

Atmospheric circulation

Weeks et al. Science 278, 1598 (1997)

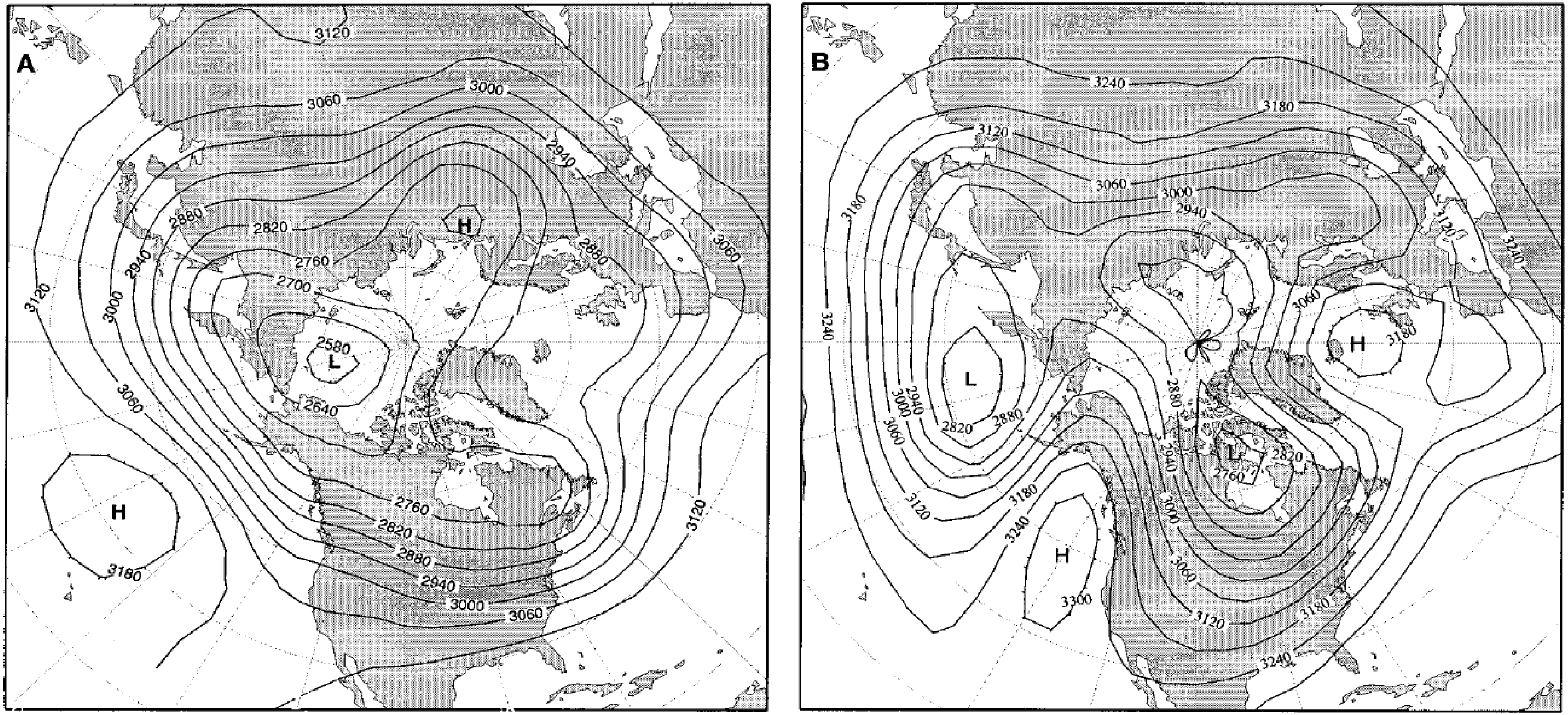
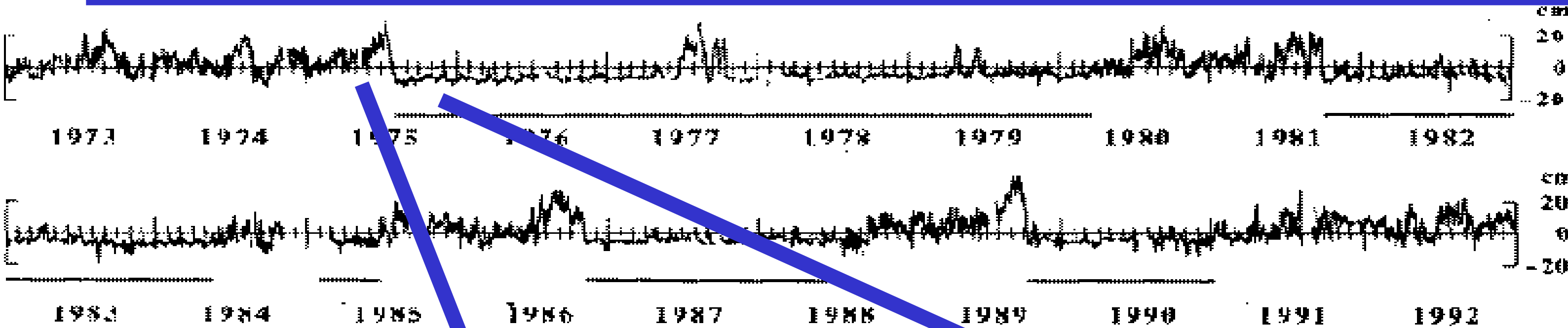


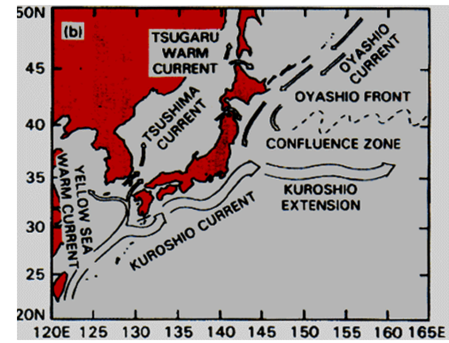
Fig. 1. Atmospheric pictures of (**A**) zonal and (**B**) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (*32*). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (*26*).

Zonal

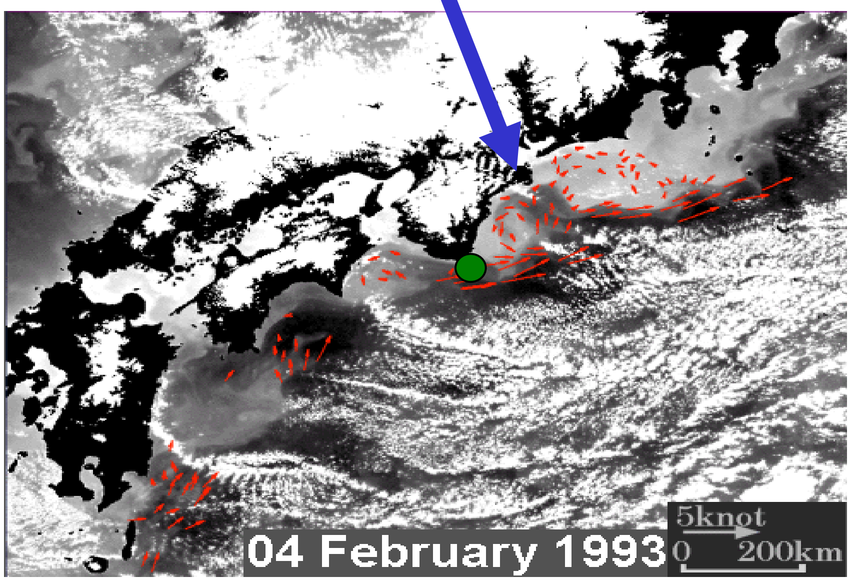
Blocked

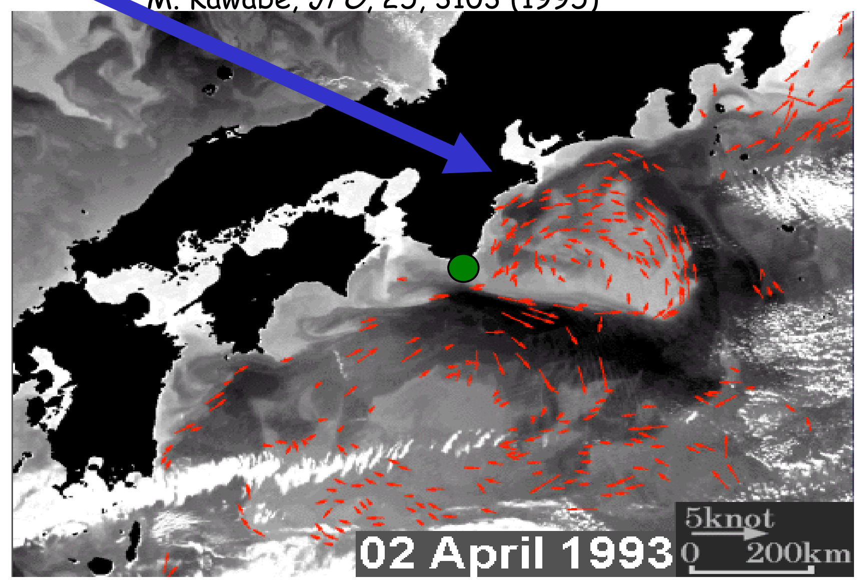
Oceanic persistent states : "Kuroshio" stream





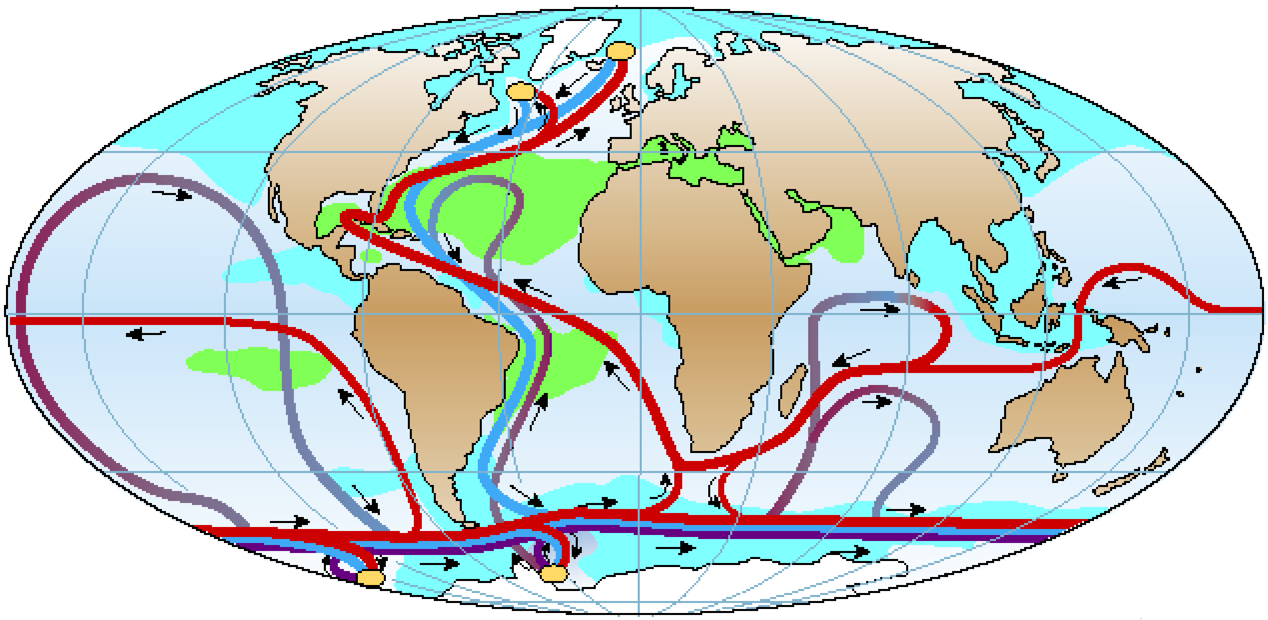
M. Kawabe, *JPO*, 25, 3103 (1995)

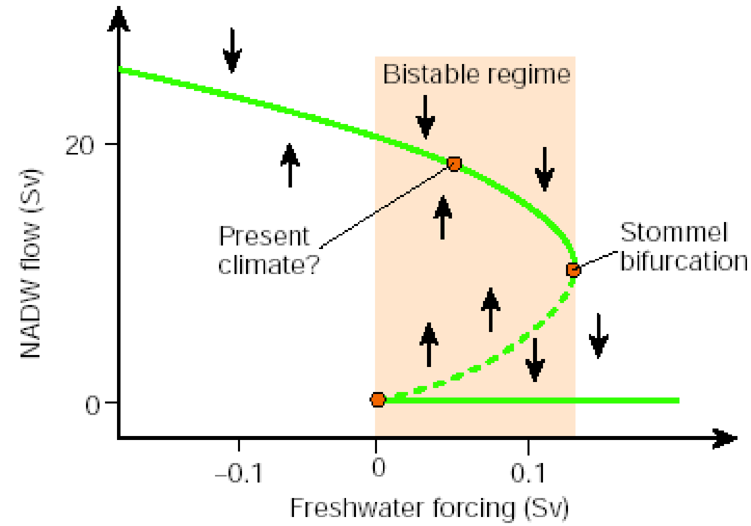




Persistent states, abrupt transitions. Scales: ~500 km, 2-5 years.
Known since 1960s (Taft 1972).

Global oceanic circulation and D/O events





Stefan Rahmstorf

NATURE | VOL 419 | 12 SEPTEMBER 2002 | www.nature.com/nature

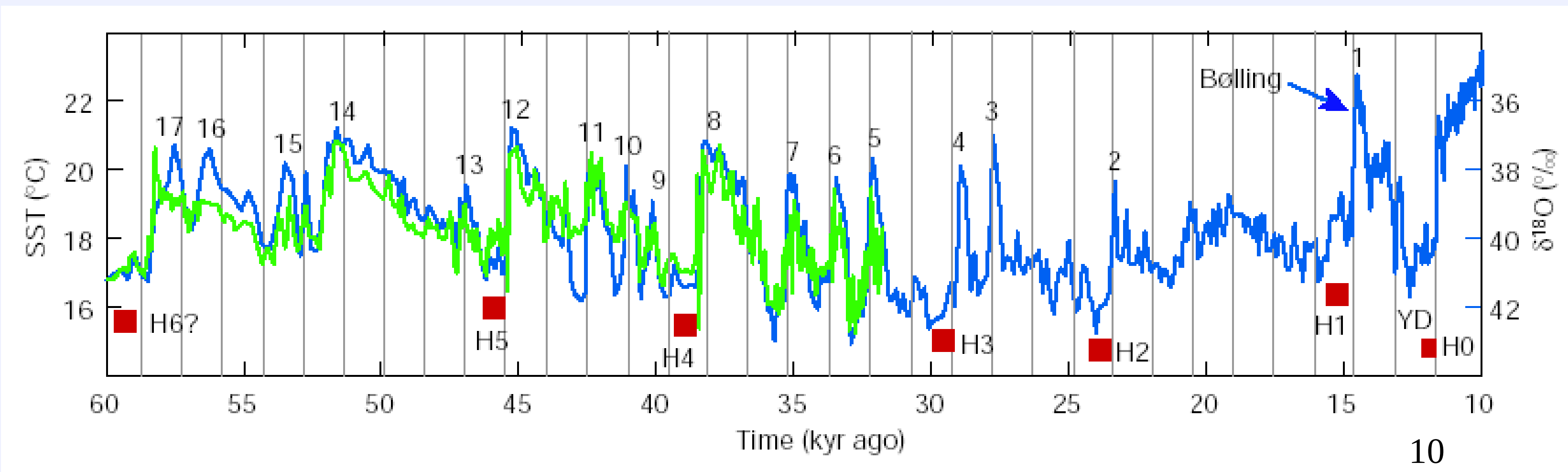
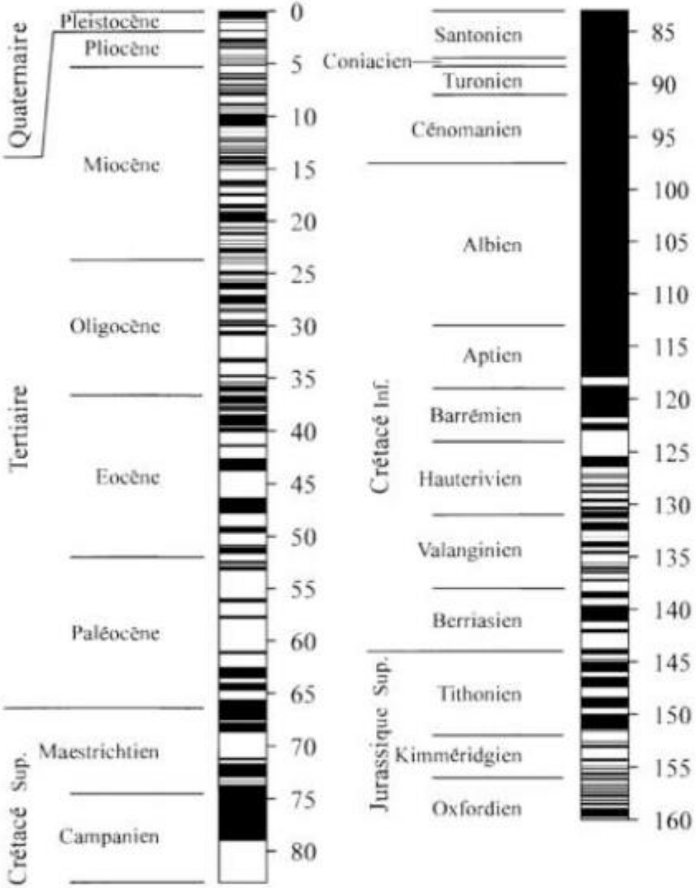
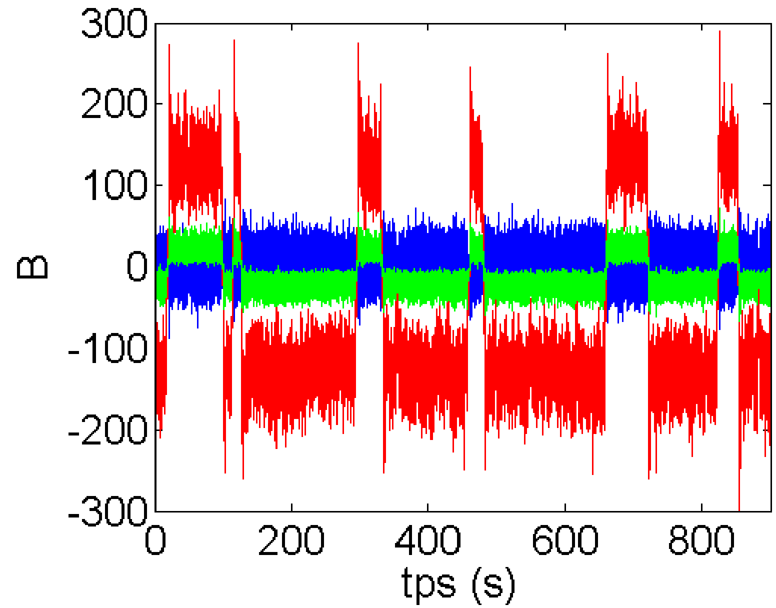


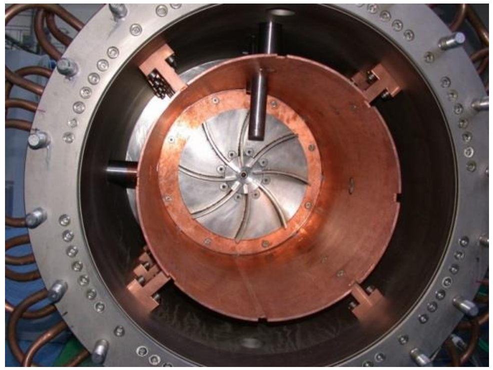
Figure 3 Temperature reconstructions from ocean sediments and Greenland ice. Proxy data from the subtropical Atlantic[86] (green) and from the Greenland ice core GISP2 (ref. 87; blue) show several Dansgaard–Oeschger (D/O) warm events (numbered). The timing of Heinrich events is marked in red. Grey lines at intervals of 1,470 years illustrate the tendency of D/O events to occur with this spacing, or multiples thereof.

Erratic inversions of the magnetic field



Earth





VKS experiment

Large scale bifurcations in turbulence

Natural systems characterized by :

- strongly turbulent flows
- several *mean large scale* states
- transitions/bifurcations *towards* or *between* these mean states on temporal scales >> fluctuations.

↓

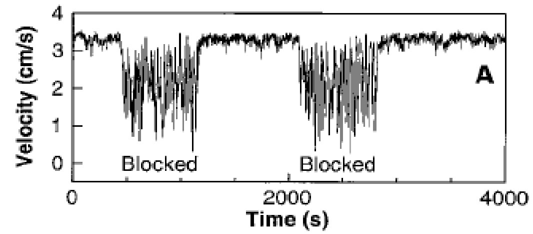
Simple model experiments:

- stability of the turbulent flow ?
- transitions:

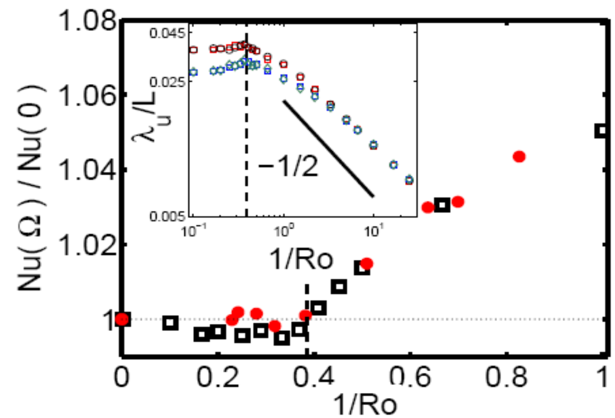
  → low dimensional dynamical systems ?

  → chaos?

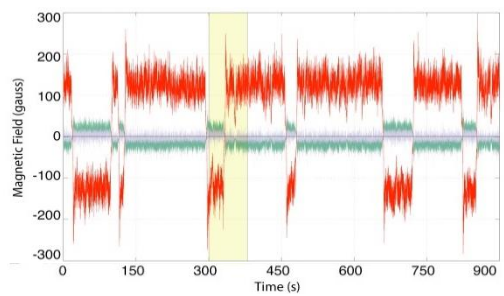
Turbulent bifurcations in lab experiments



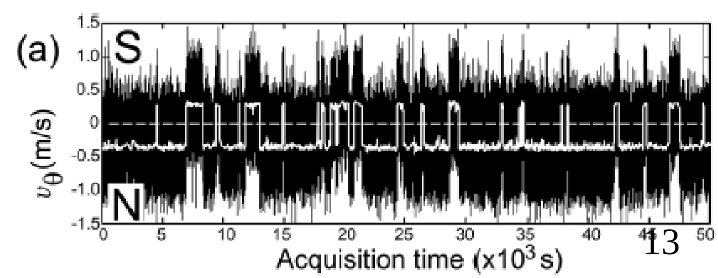
- *geophysical experiments:*
  Weeks et al. Science (1997)
- *RB convection at high Ra* :
  Chilla et al., EPJB (2004) - Roche et al. NJP (2010) - Grossmann and Lohse, POF (2011) - Alhers et al. NJP (2011) - van der Poel et al. (2011)
- *turbulent rotating RB convection:*
  Stevens et al. PRL (2009) - Weiss et al., PRL (2010)



- *spherical Couette flow*
  Zimmermann et al. (2011)



- *Taylor-Couette flow*
  Lathrop et Mujica (2006)
- *von Karman flows*
  VKS (2007)
  de la Torre and Burguete (2007, 2012)
- *wake behind a sphere*
  Cadot et al. (2013)
- ……



Turbulence or chaos?

However

- no complete theory of these transitions
- all tentative to find the *strange attractor* of a turbulence state (atmosphere or climate) failed so far:
  - Nicolis 84: yes
  - Grassberger 86: no
  - Lorenz 91: unlikely!

I therefore see no reason to believe that an extensive weather or climate system possesses a low-dimensional attractor. At the same time, I do not feel that most of the real-data studies are meaningless; they merely need to be reinterpreted. As suggested in one study[5], the atmosphere might be viewed as a loosely coupled set of lower-dimensional subsystems. Perhaps the procedure, as practised, attempts to measure the dimension of a subsystem. □

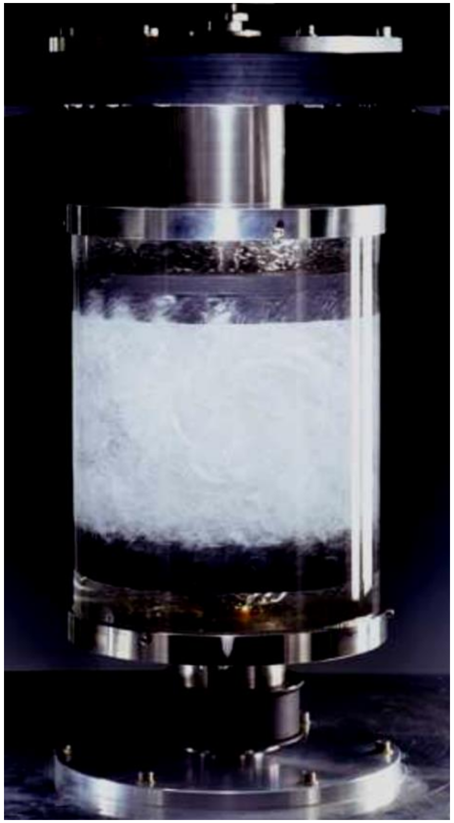
Turbulence or chaos?

End of the story?
abandon all hope to apply tools from dynamical systems theory to a turbulent flow, except for transition to turbulence?

No!

→ in this talk:
try to reconcile the 2 points of view in fully developed turbulence

A model experiment: the von Karman flow

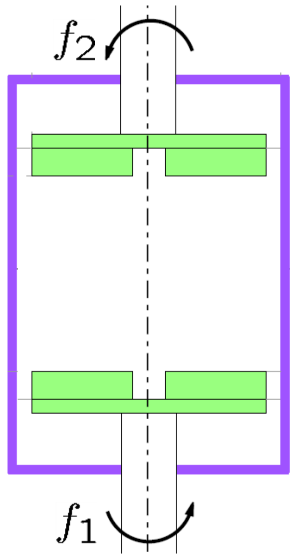


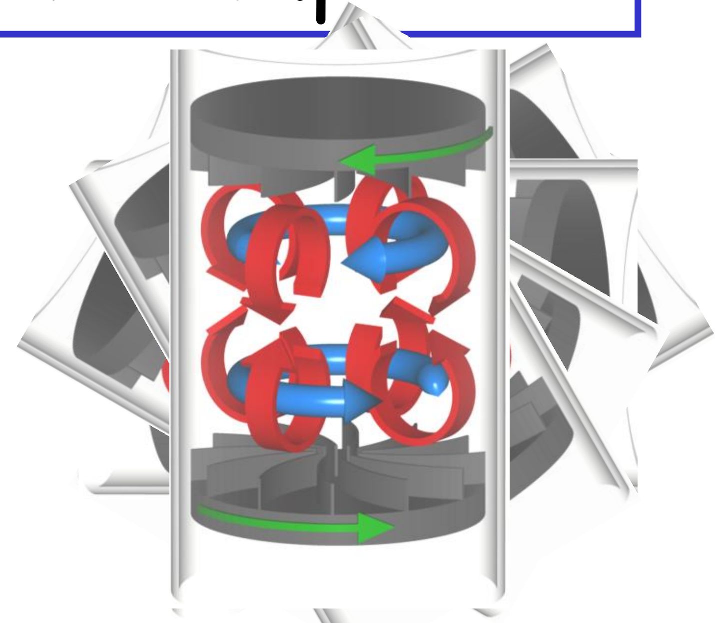
- flow between 2 rotating impellers
- *Re* = 10^2 to 10^6 (water and water-glycerol)
  10^7 (Sodium) and 10^8 (liquid Helium)

- inhomogeneous and intense turbulence

- different forcings:

Experimental setup





symétrie $\mathcal{R}_\pi$

2 control parameters

Reynolds number

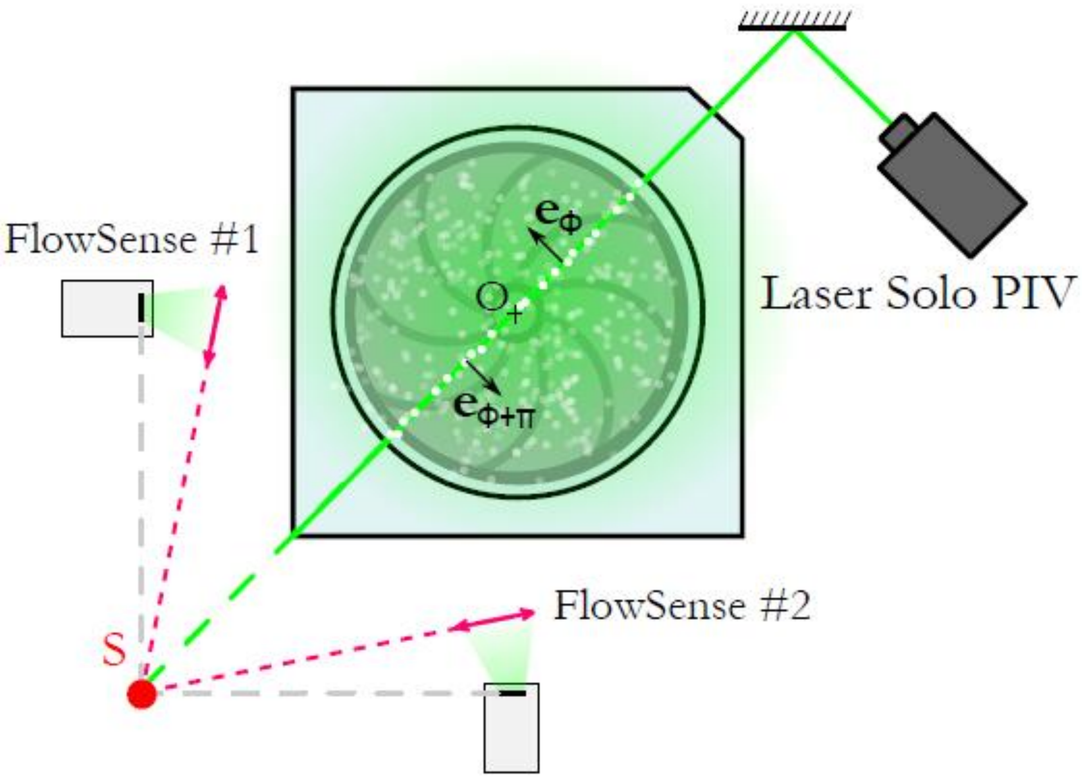
Rotation number
(System asymmetry)

$$Re = \frac{2\pi f R^2}{\nu}$$

$$f = \frac{f_1 + f_2}{2}$$

$$\theta = \frac{f_1 - f_2}{f_1 + f_2}$$

Measurements



- SPIV

→3 velocity components in a vertical plane

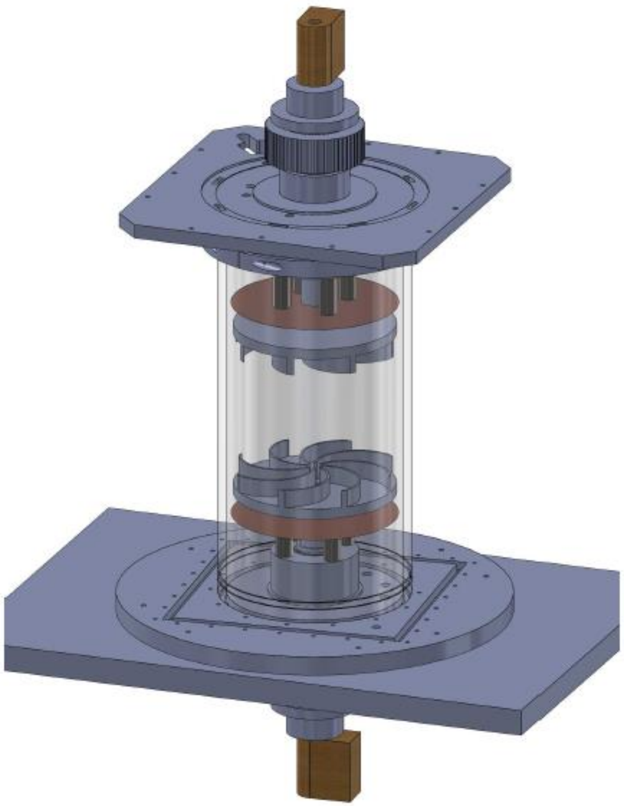
→ up to 50 000 fields

→ freq max=15 Hz

→ resolution = 2 mm

- Torque measurements

Transition to turbulence in von Karman



Re ↓

laminar

stationary pattern

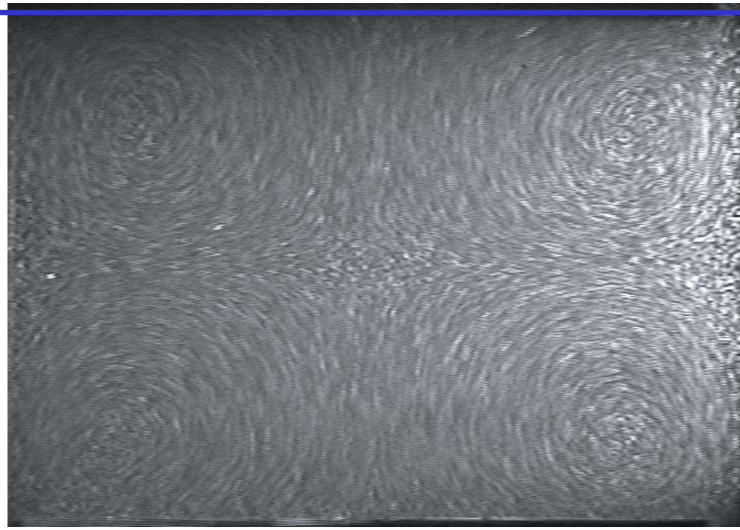
oscillating pattern

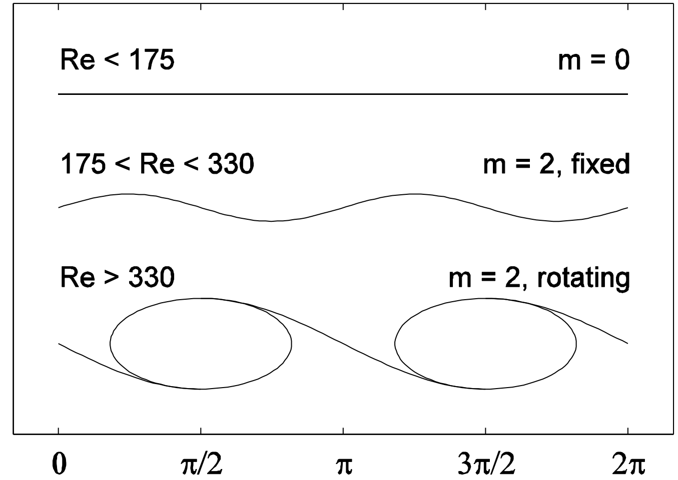
chaos

turbulence

First bifurcations and symmetry breaking

meridian plane: poloïdal recirculation



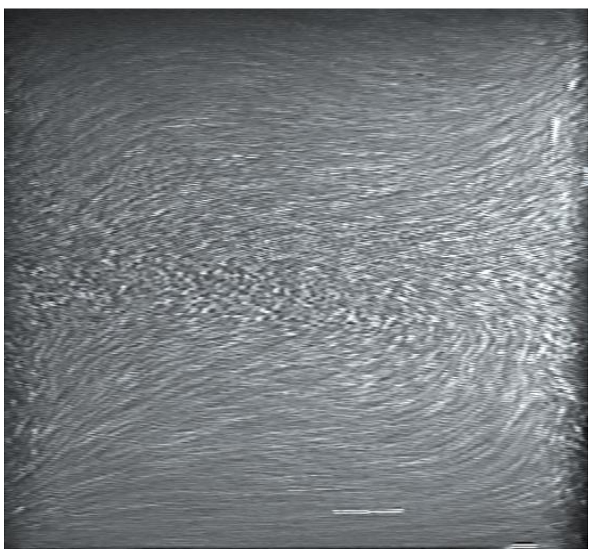


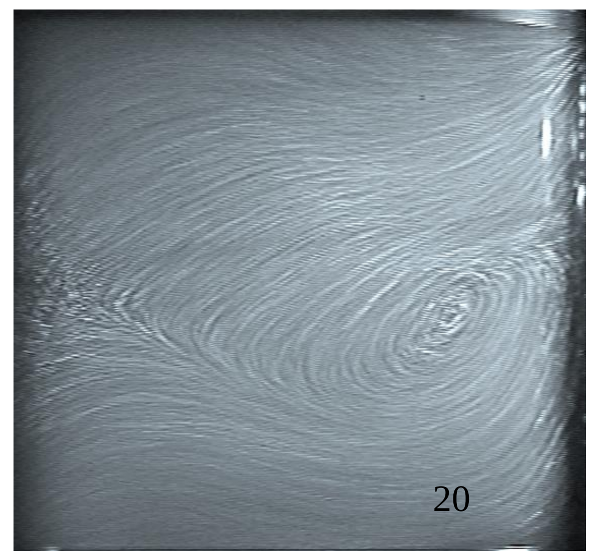
Re = 90
Stationary axisymmetric

Re = 185
m = 2 ; stationary

Re = 400
m = 2 ; periodic

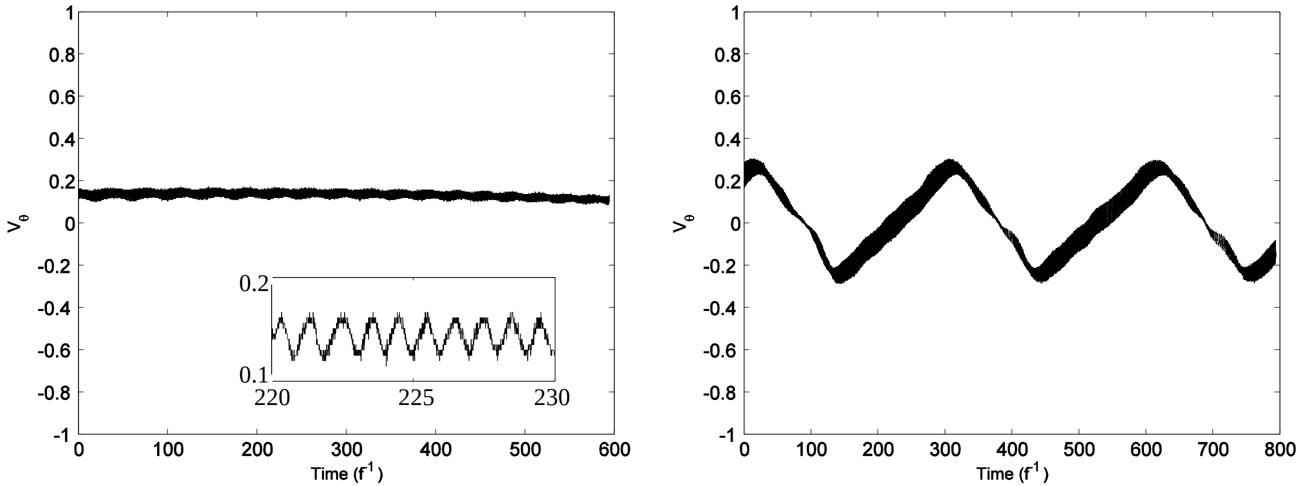
Tangent plane : shear layer



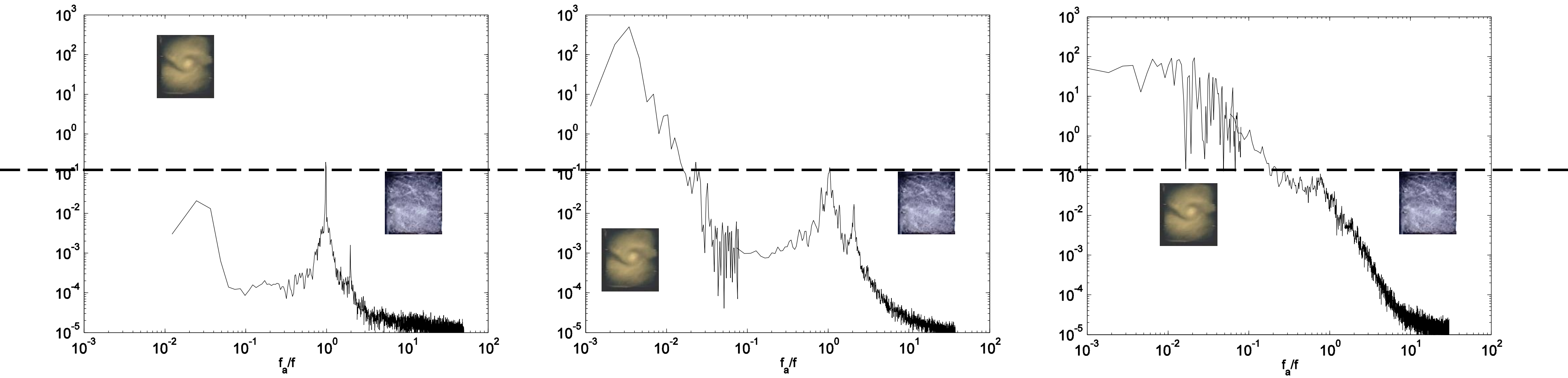


Time spectra as a function of Re

v_θ en $\{r = 0.9; z = 0\}$



Re = 330	**Re = 380**	**Re = 440**
Periodic	Quasi-Periodic	Chaotic



Time spectra as a function of Re

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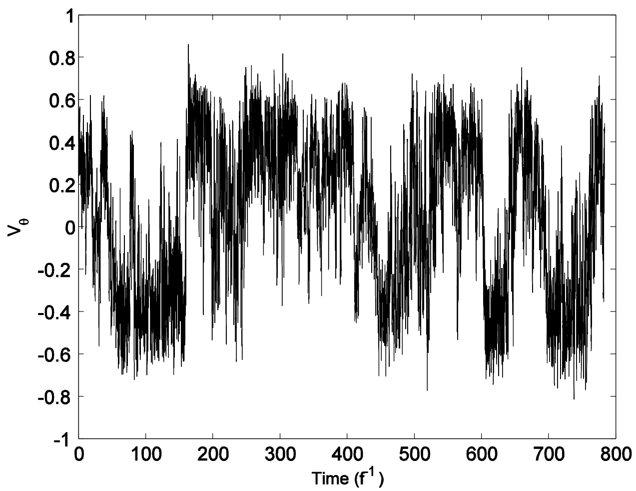
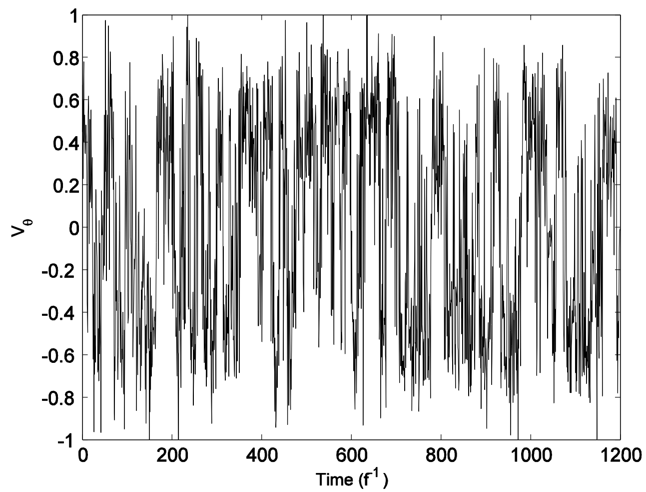
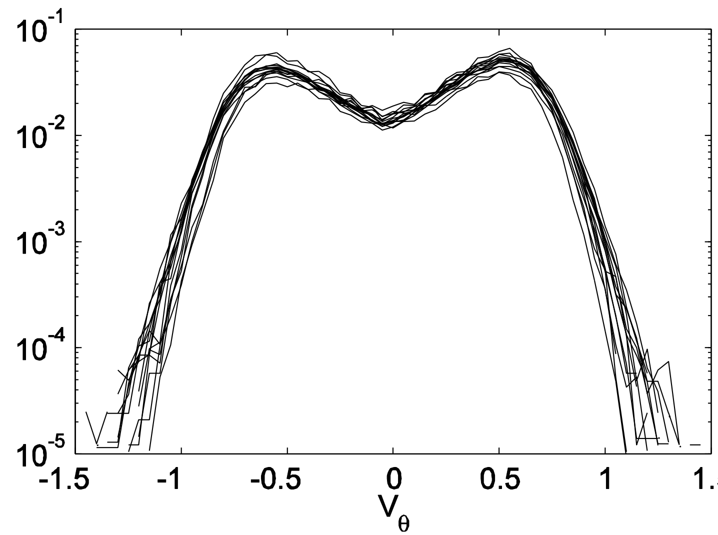


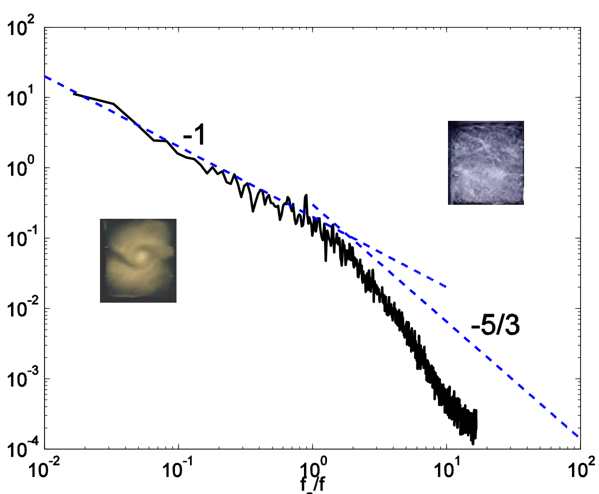


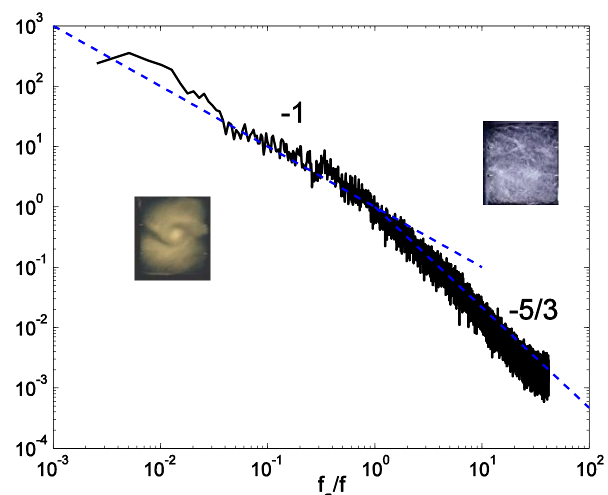


Re = 1000
Chaotic

Re = 4000
Turbulent

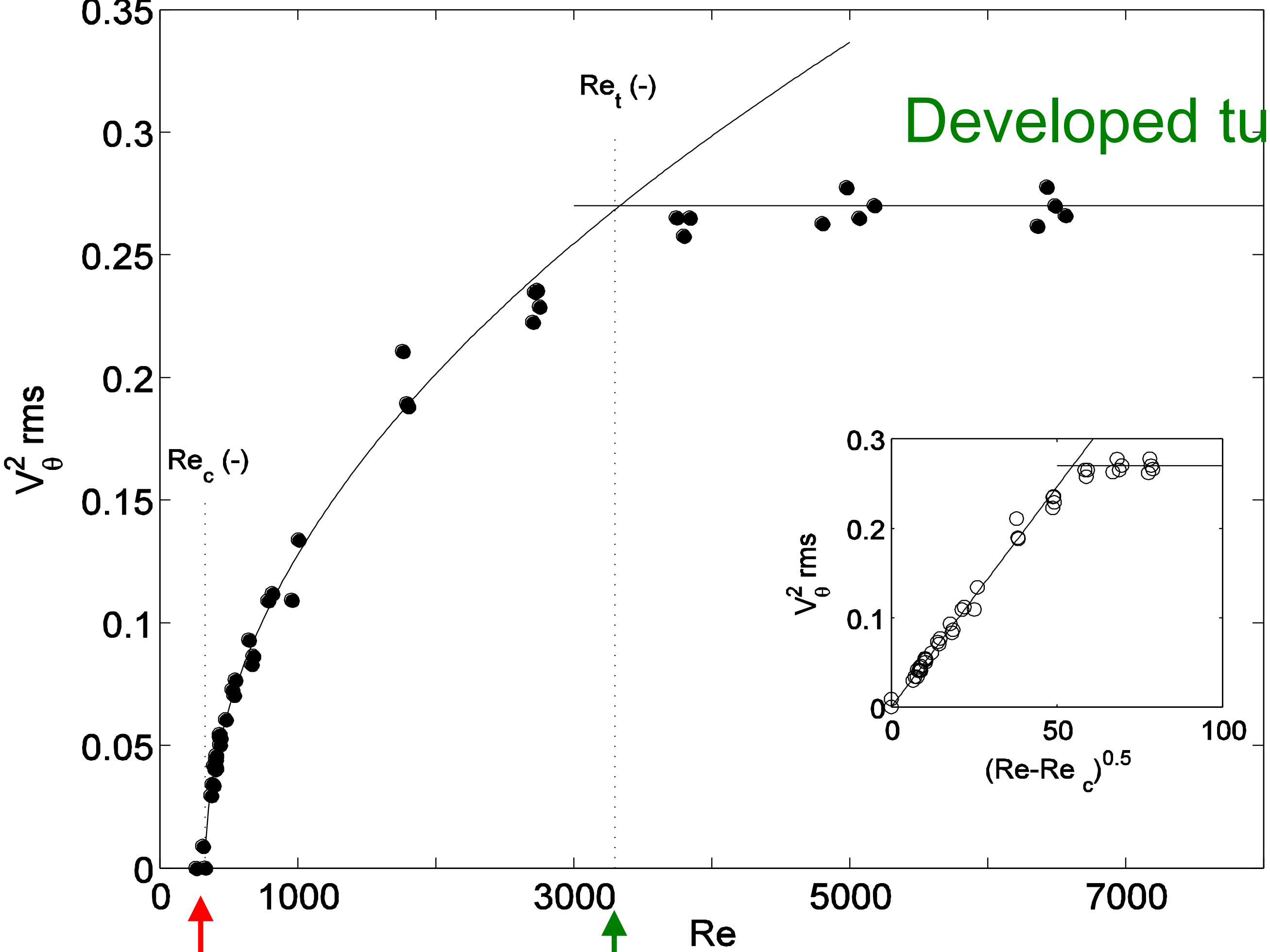
2000 < Re < 6500





Bimodal distribution :
signature of the turbulent shear

Transition to turbulence: azimuthal kinetic energy fluctuations



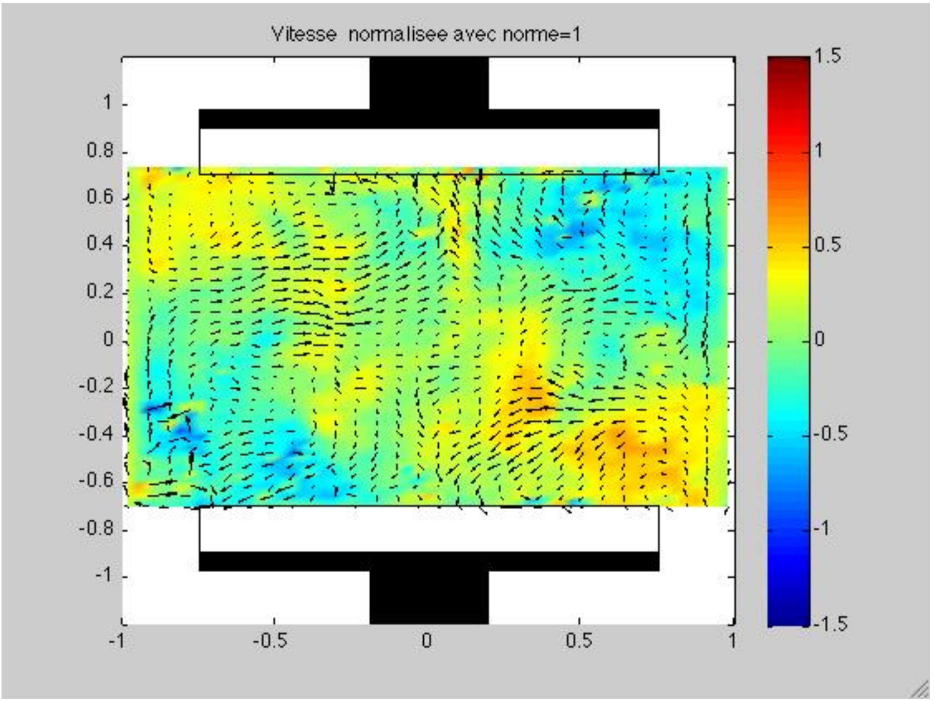
Developed turbulence

Globally supercritical transition via a Kelvin Helmholtz type instability of the shear layer and secondary bifurcations

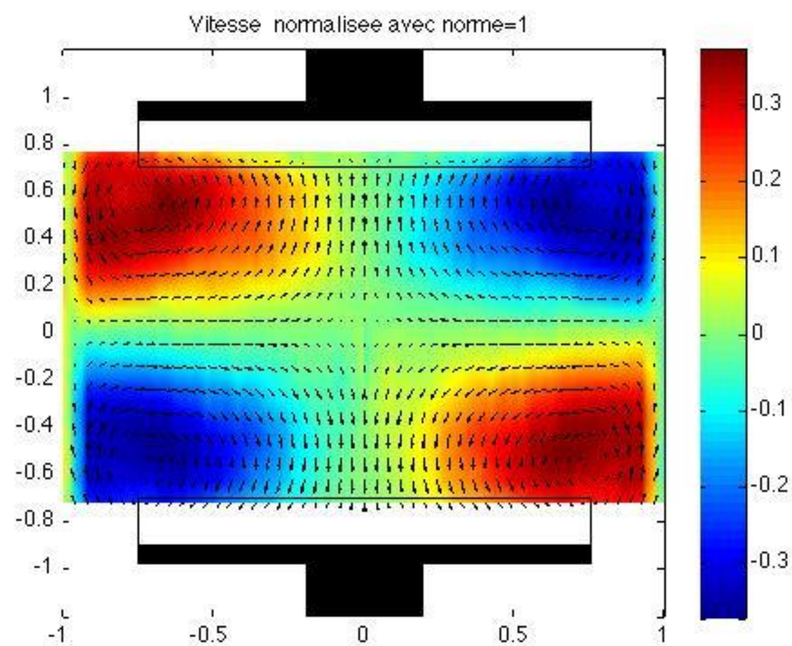
$Re_c = 330$ $Re_t = 3300$

Ravelet et al. JFM 2008

Turbulent von Karman at Re> 5000

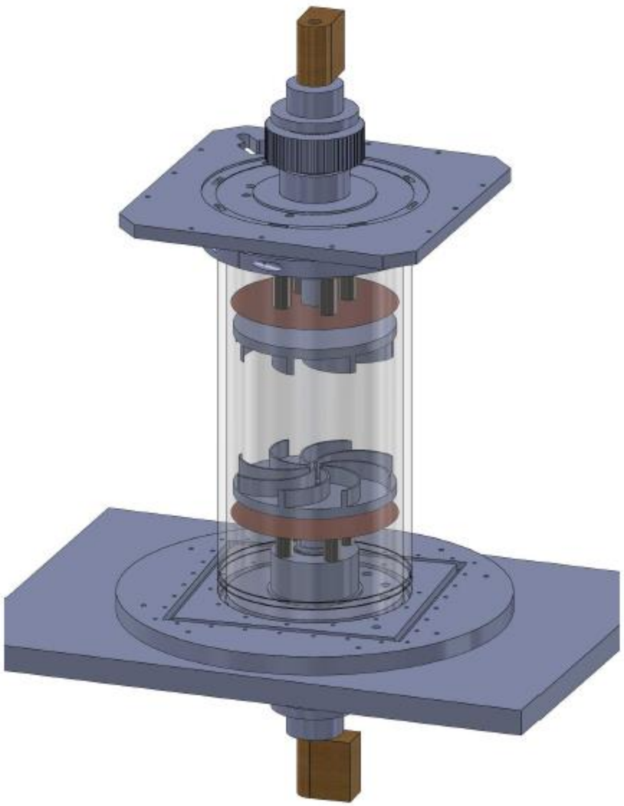


Instantaneous Flow



Mean flow

Transition to turbulence in von Karman



Re

laminar

stationary pattern

oscillating pattern

chaos

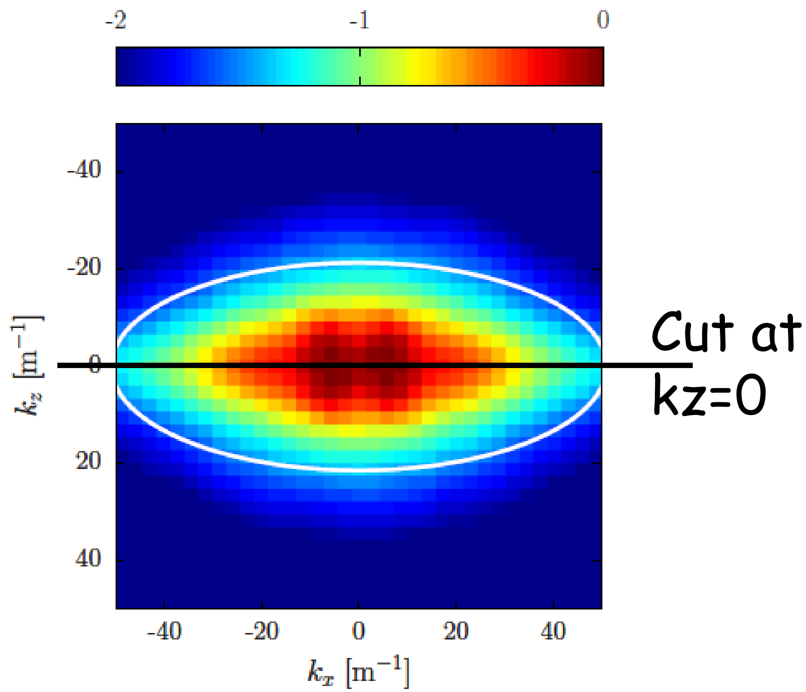
turbulence

……

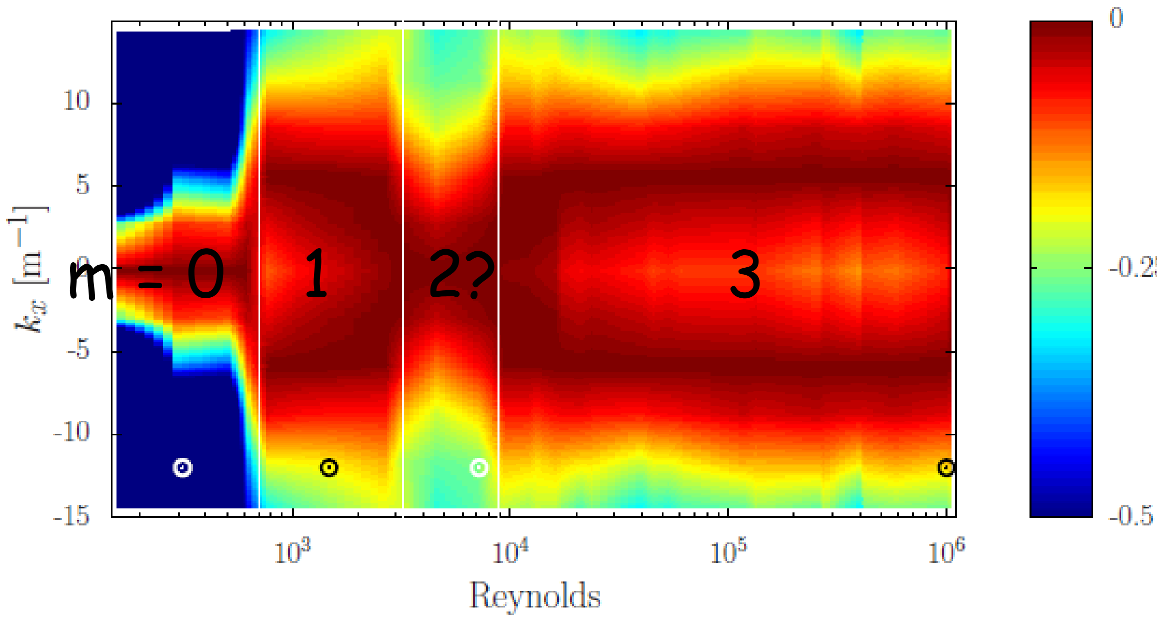
End of story?

Eckhaus type instability (1)

+ sense, fully turbulent regime, Kp constant, and yet…



Energy spectrum
(Re= 10^6)



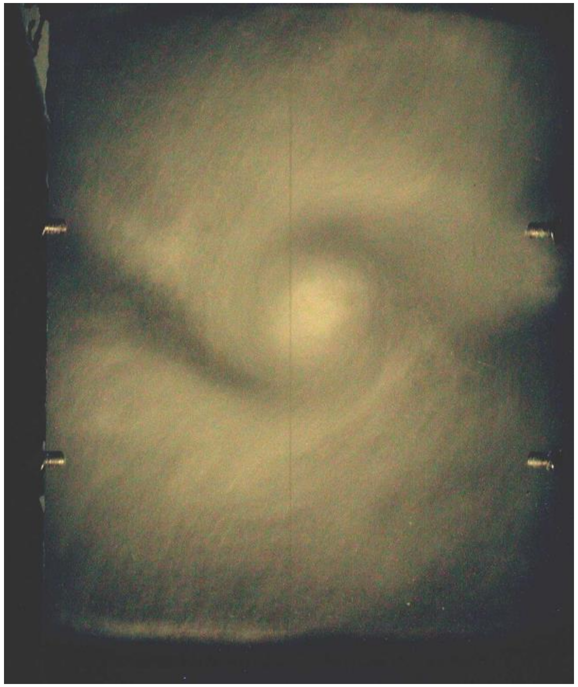
Evolution of k_x with Re

Herbert et al. Phys. Fluids 2013

Eckhaus type instability (2)

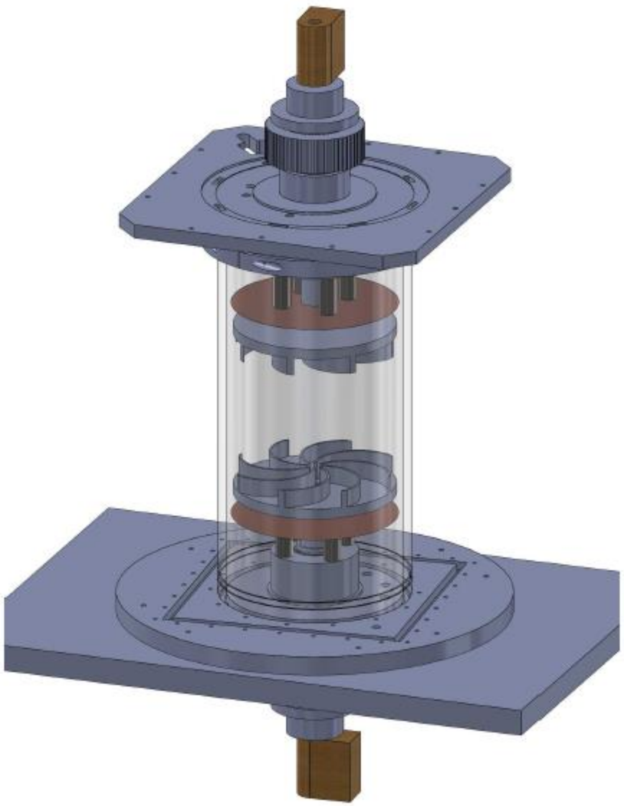
→ parity change (m change)

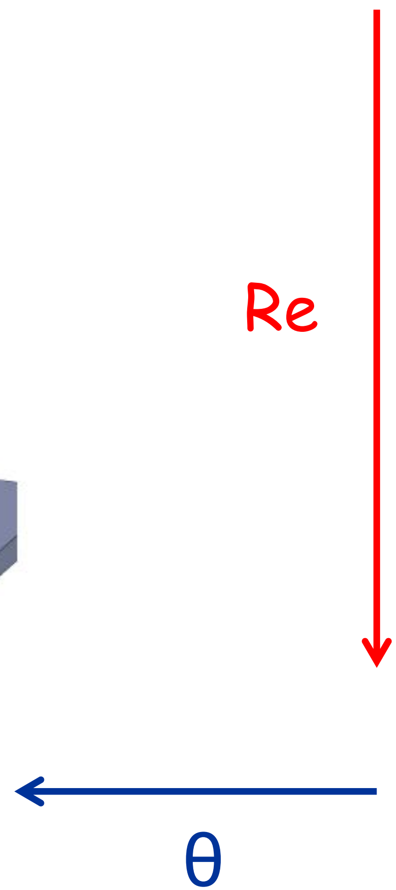
- cf. laminar/turbulent transition
- related to the number of mean turbulent vortices in the shear layer



Re= 10^4: von Karman turbulent flow « stable » (no m change up to Re = 10^6) …

Transition to turbulence in von Karman





laminar

stationary pattern

oscillating pattern

chaos

turbulence

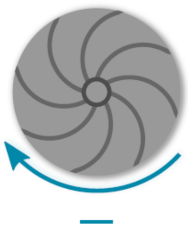
Eckhaus type instability

turbulence*

End of story?

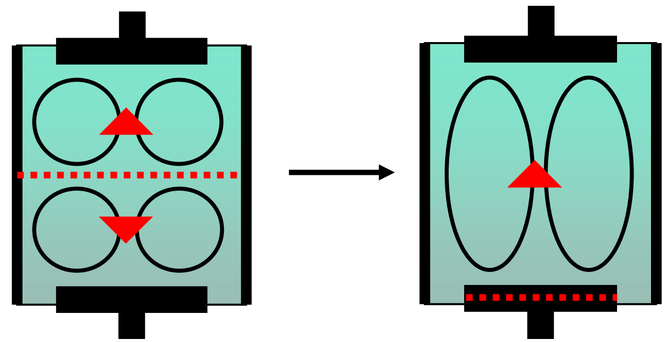
Transition of the turbulent mean state

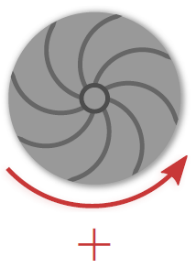
→ response to an external symmetry breaking: θ≠0



- sense -: abrupt transition

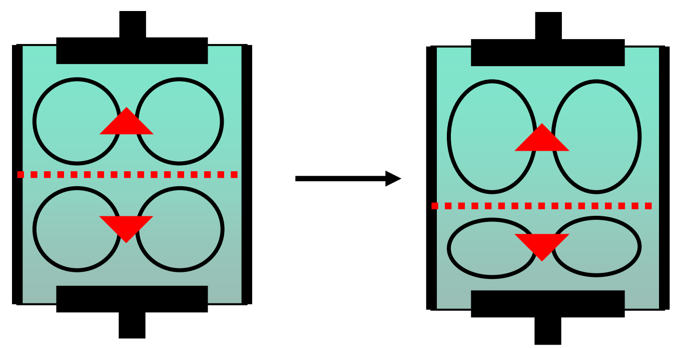
Ravelet et al. Phys. Rev. Lett. 2004
Saint-Michel et al. Phys. Rev. Lett. 2013
NJP 2014
Thalabard et al. NJP 2015



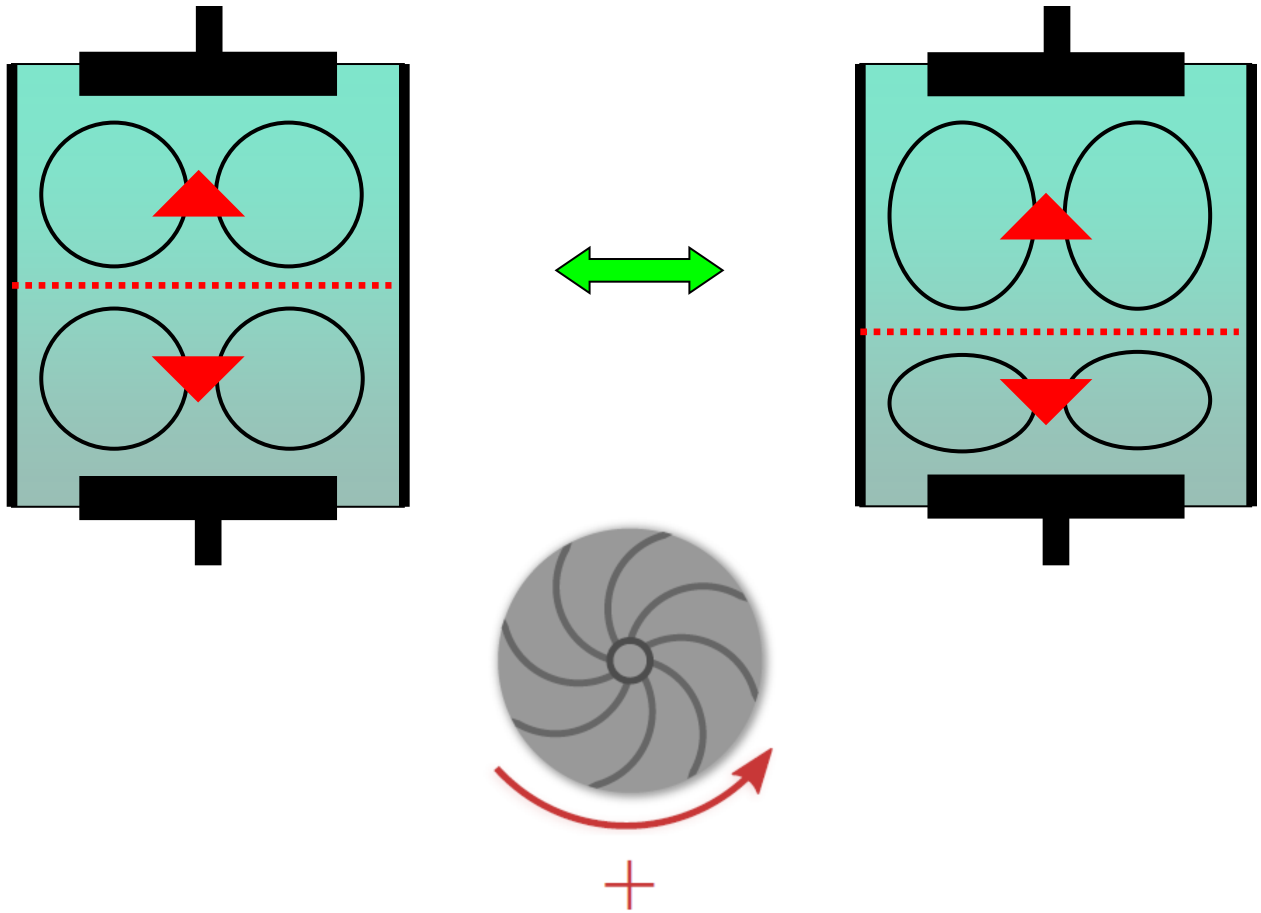


- sense +: soft transition

Cortet et al. Phys. Rev. Lett. 2010
J. Stat. Mech. 2011



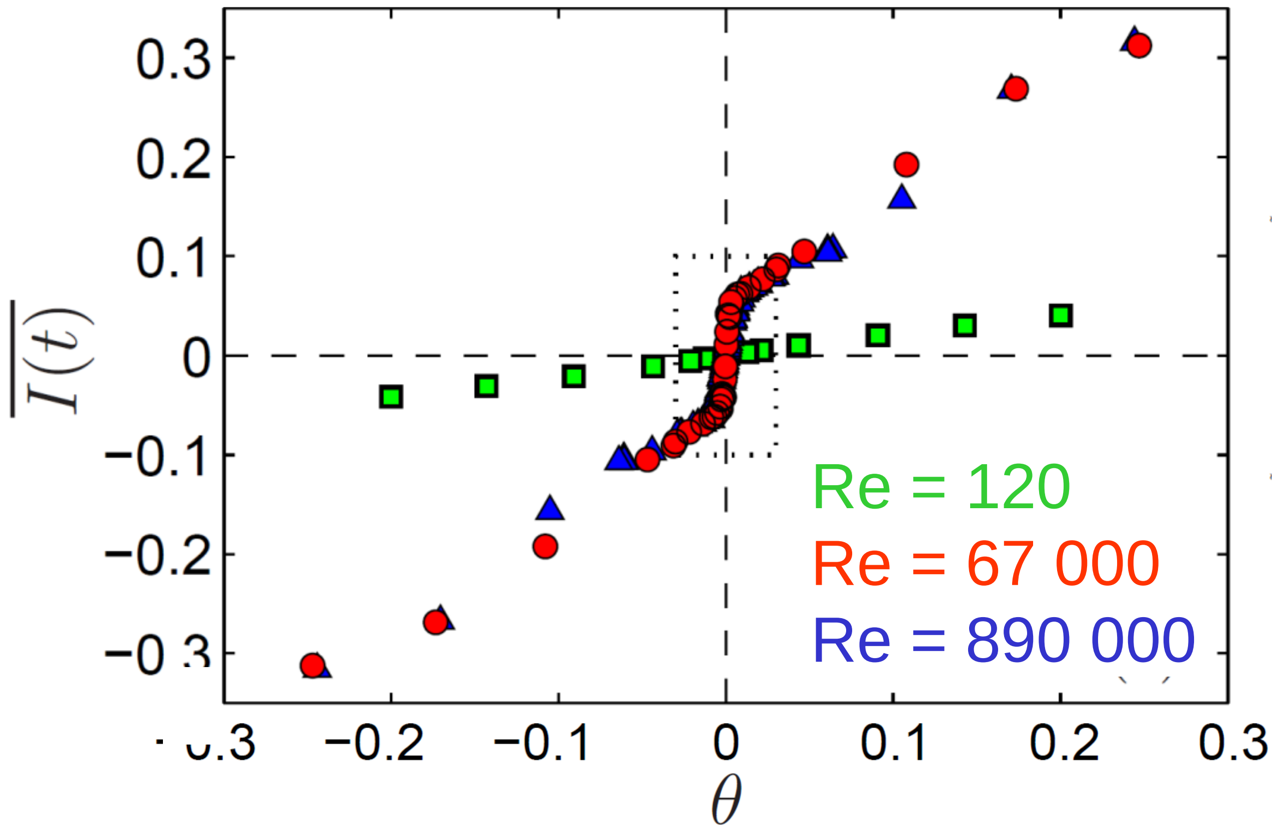
Soft transition

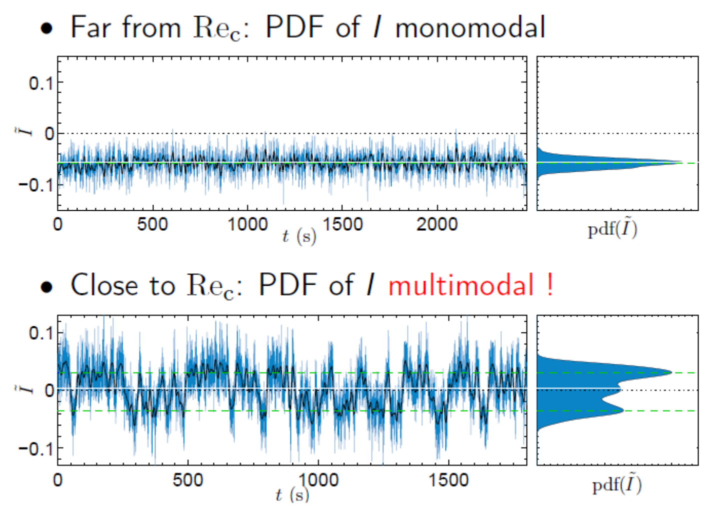


Global angular momentum I = f(Θ)

Order parameter: kinetic momentum

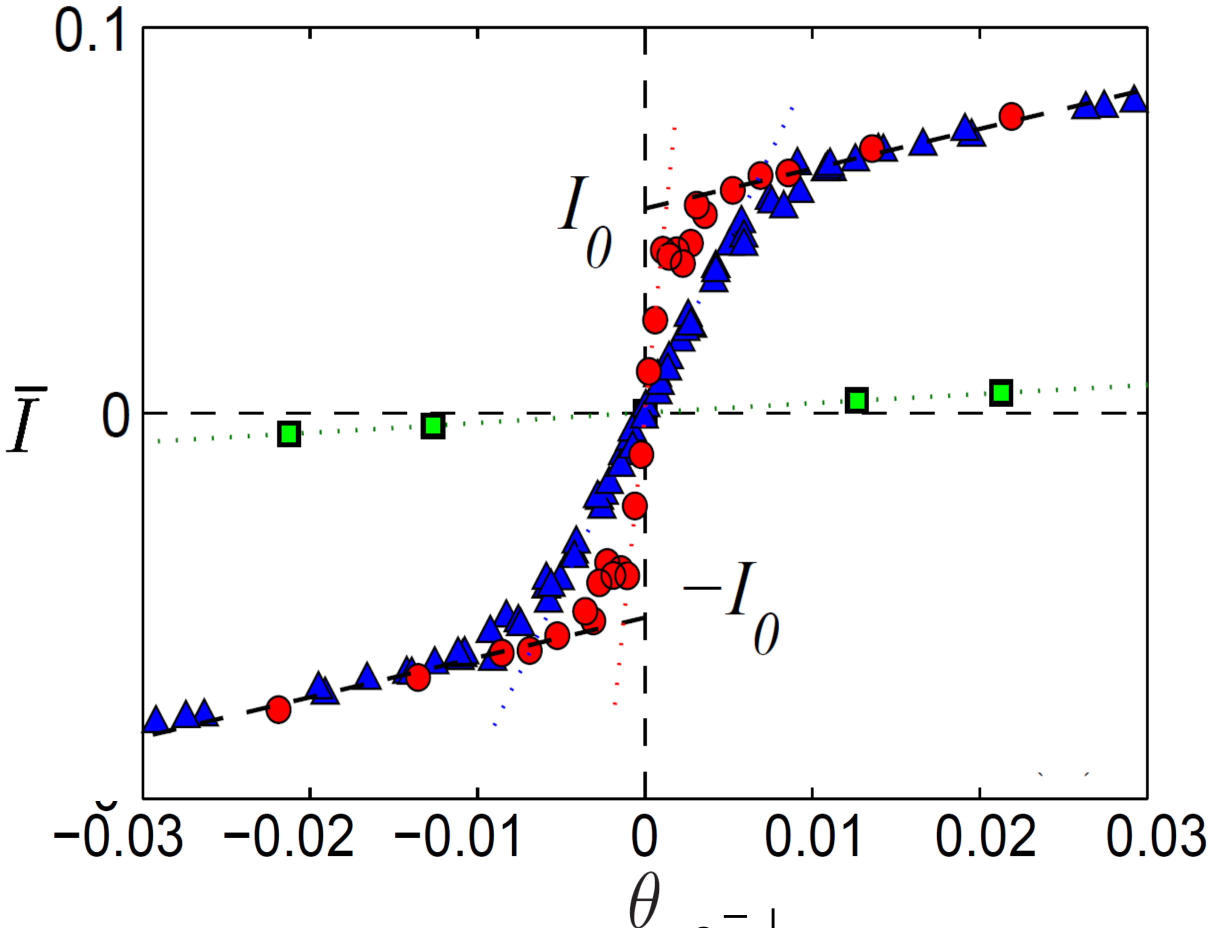
$$I(t) = \frac{1}{V}\int r dr d\varphi dz \frac{r u_\varphi(t)}{\pi . R^2 (f_1 + f_2)}$$





Symmetry parameter: $\theta = \frac{f_1 - f_2}{f_1 + f_2}$

Global angular momentum I = f(Θ)



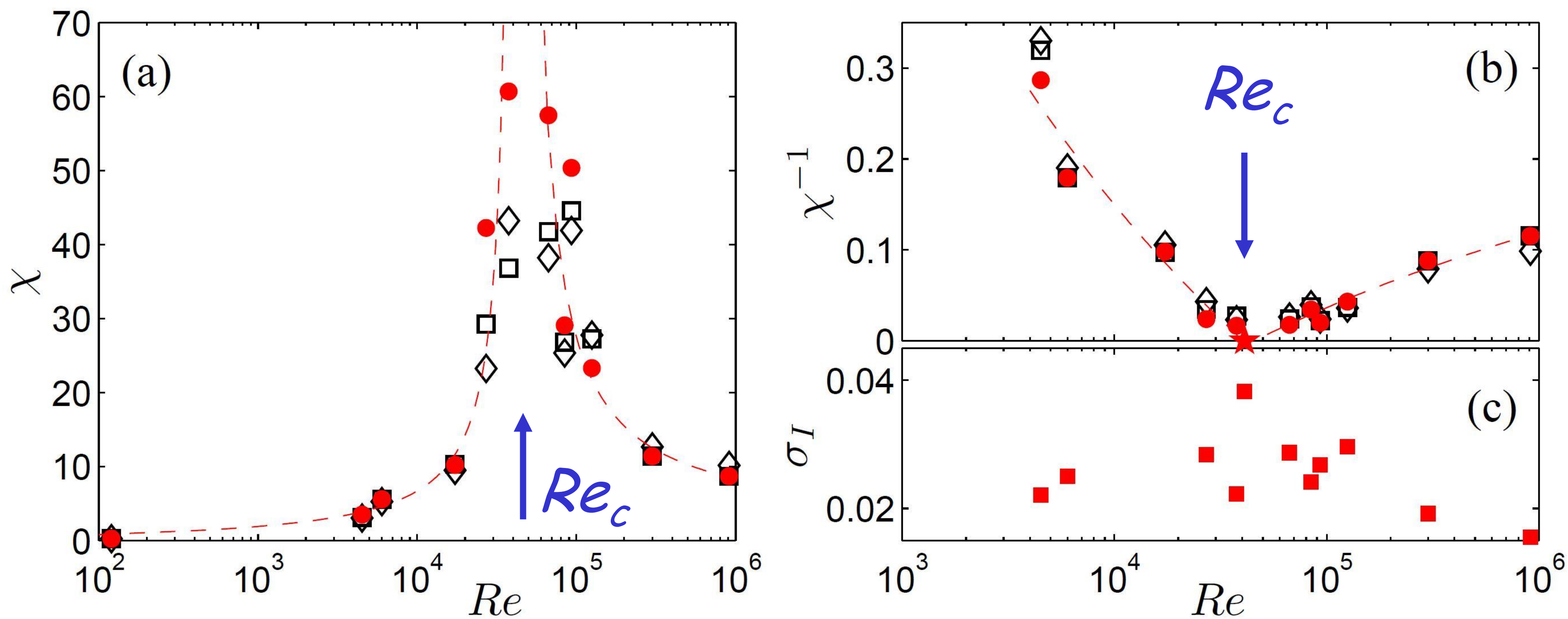
Re = 120 → $\chi_I = 0.24$

Re = 67 000 → $\chi_I = 42$

Re = 890 000 → $\chi_I = 9$

Susceptibility $\chi_I = \left.\frac{\partial \bar{I}}{\partial \theta}\right|_{\theta=0}$ larger at intermediate Re

Susceptibility: χ = f(*Re*)



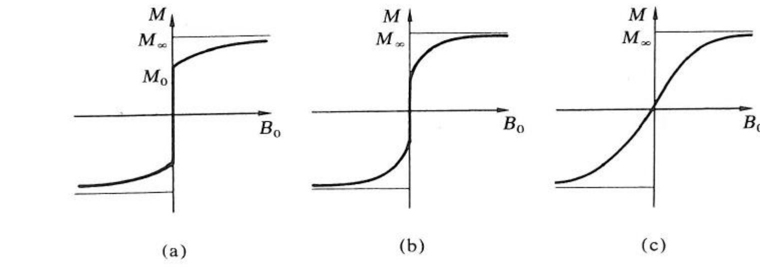
Divergence of the susceptibility around $Re = Re_c \approx 40\,000$

→ *phase transition?*

Analogy with ferromagnetic systems

Magnetization M ⟷ angular momentum I
Applied field H ⟷ symmetry param. θ
Temperature T ⟷ Reynolds number Re

$$\chi = \frac{\partial M}{\partial H} \Leftrightarrow \chi = \frac{\partial I}{\partial \theta}$$



T<Tc T=Tc T> Tc

Magnetization

Control parameter: turbulence « temperature »

$$T \sim 1/\log Re$$

Castaing, J. Phys. II (1996)

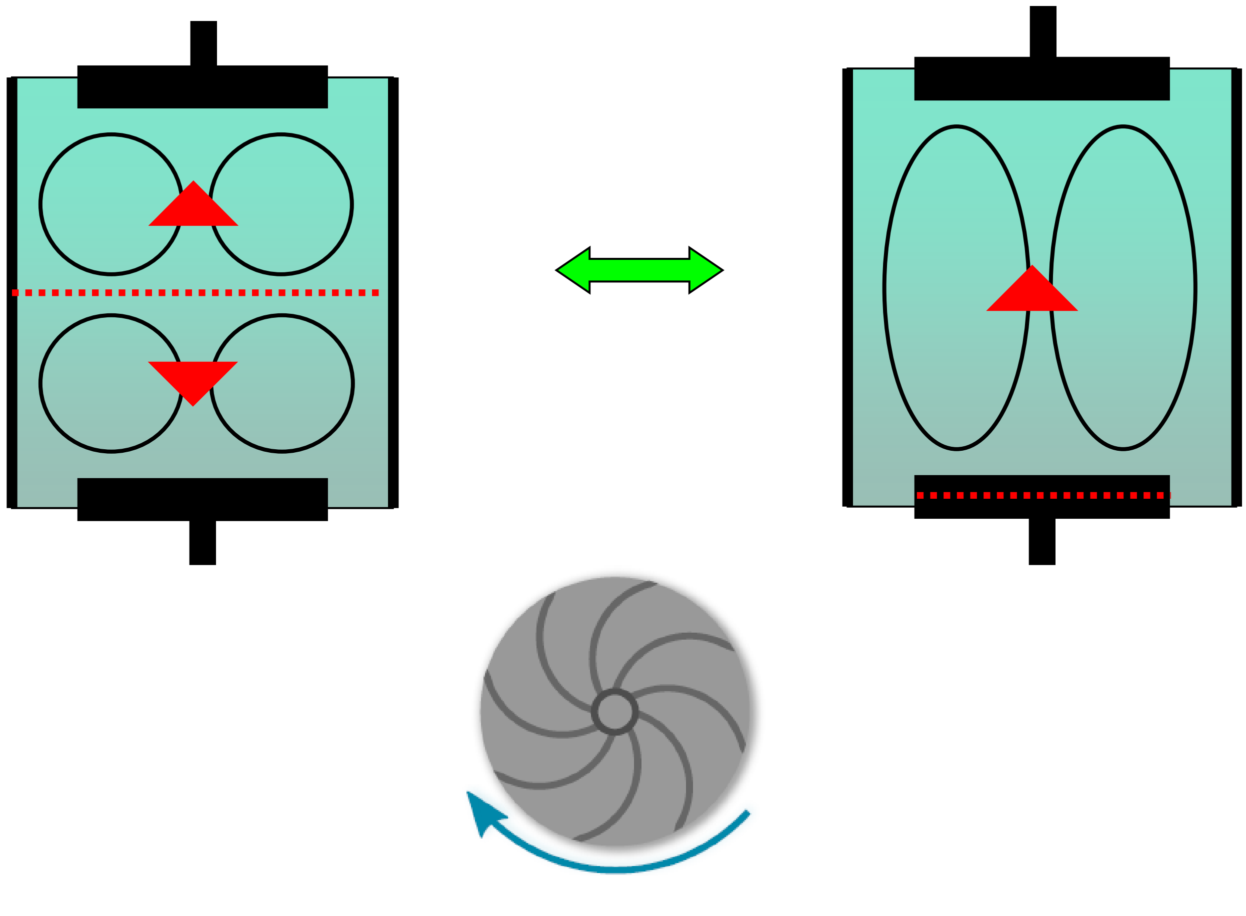
Mean field critical divergence

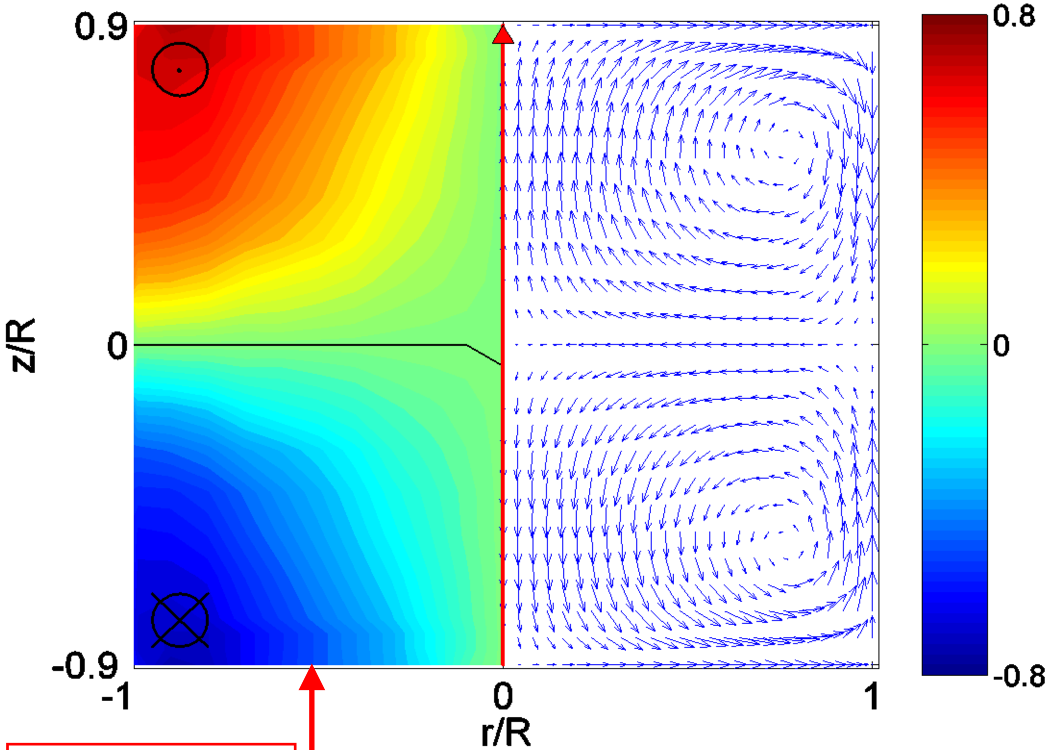
$$\chi \propto |T - T_c|^{-1}$$

with Re_c = 40 000

$$\chi \propto \left|\frac{1}{\log Re} - \frac{1}{\log Re_c}\right|^{-1}$$

Abrupt transition

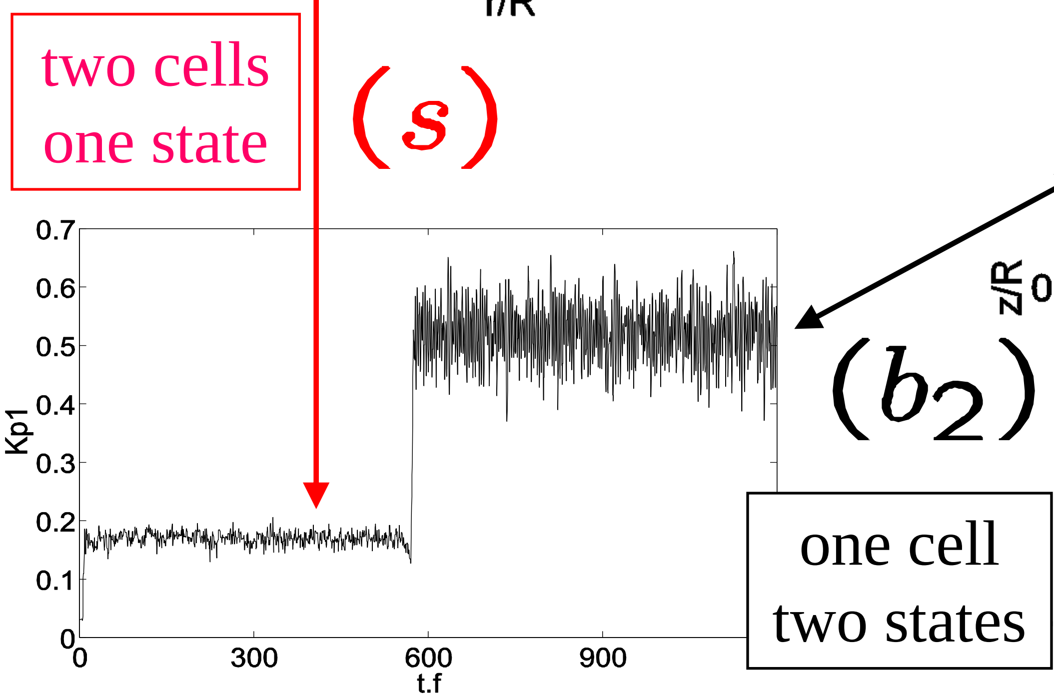


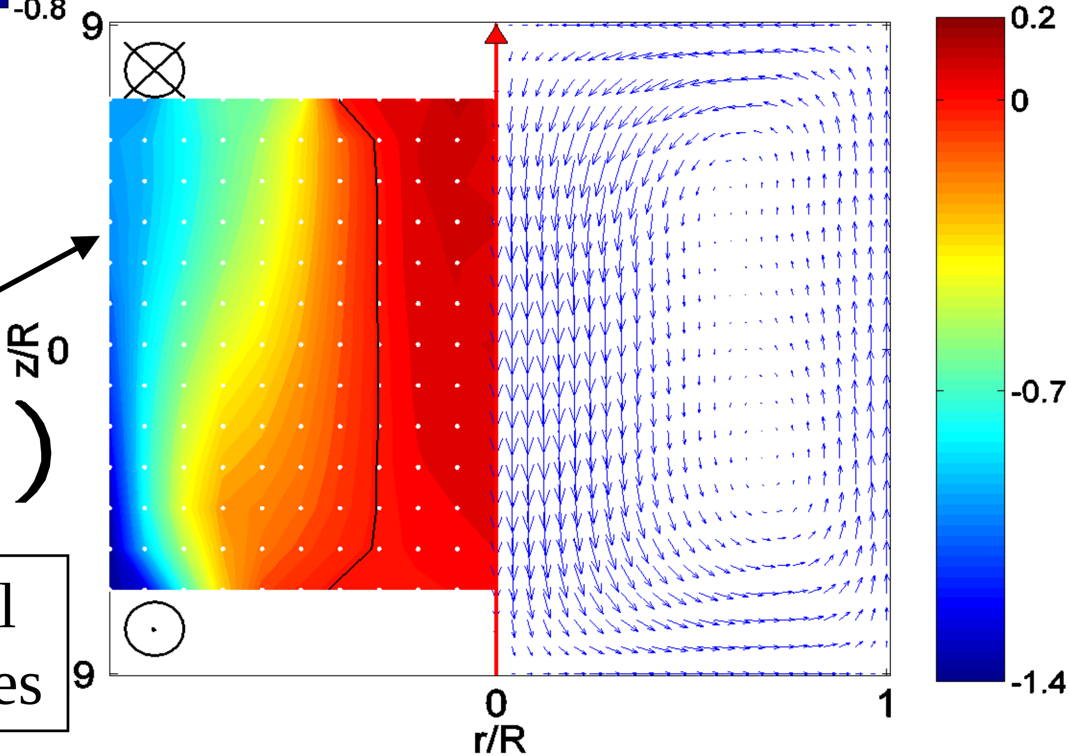


Turbulent Bifurcation:

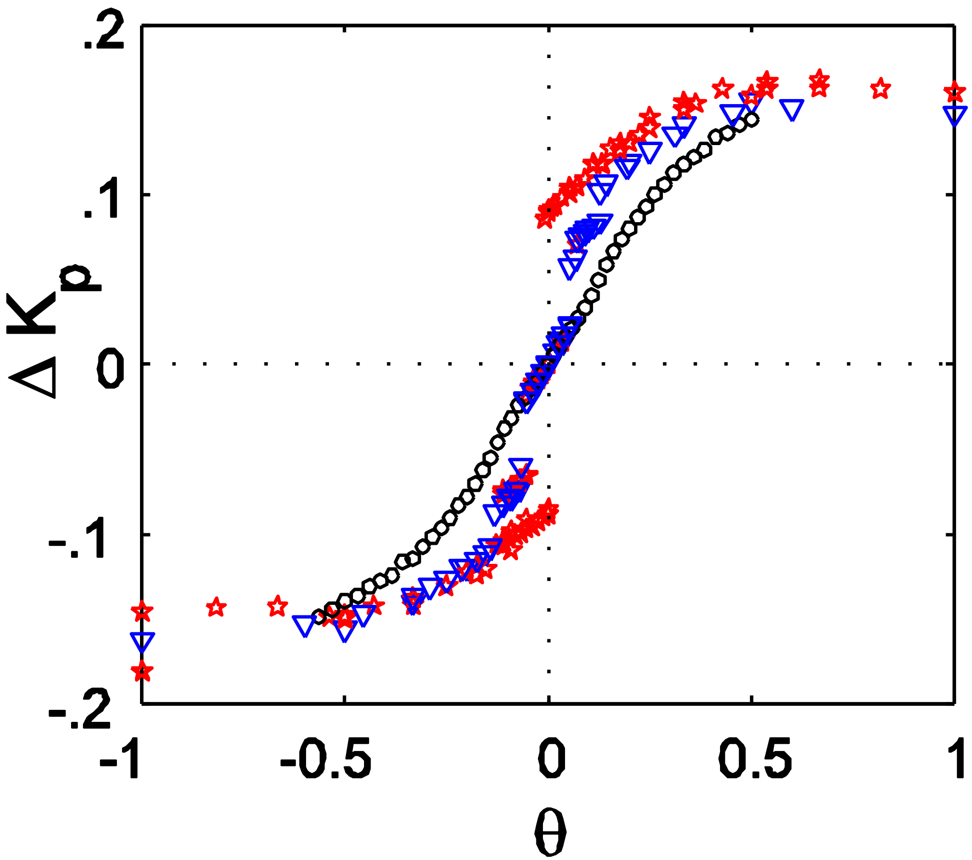
2 different mean flows exchange stability.
A symmetry is broken

Bifurcated flow (b) : no more shear layer

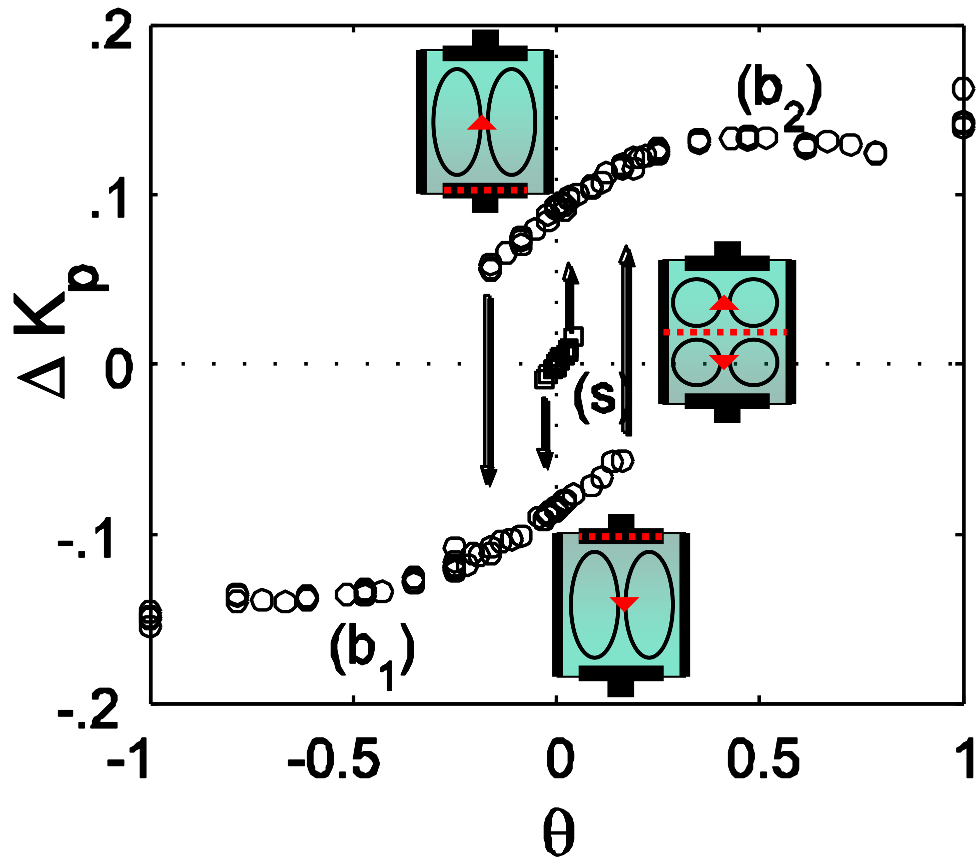




coexistence of the 3 states only in turbulent regimes

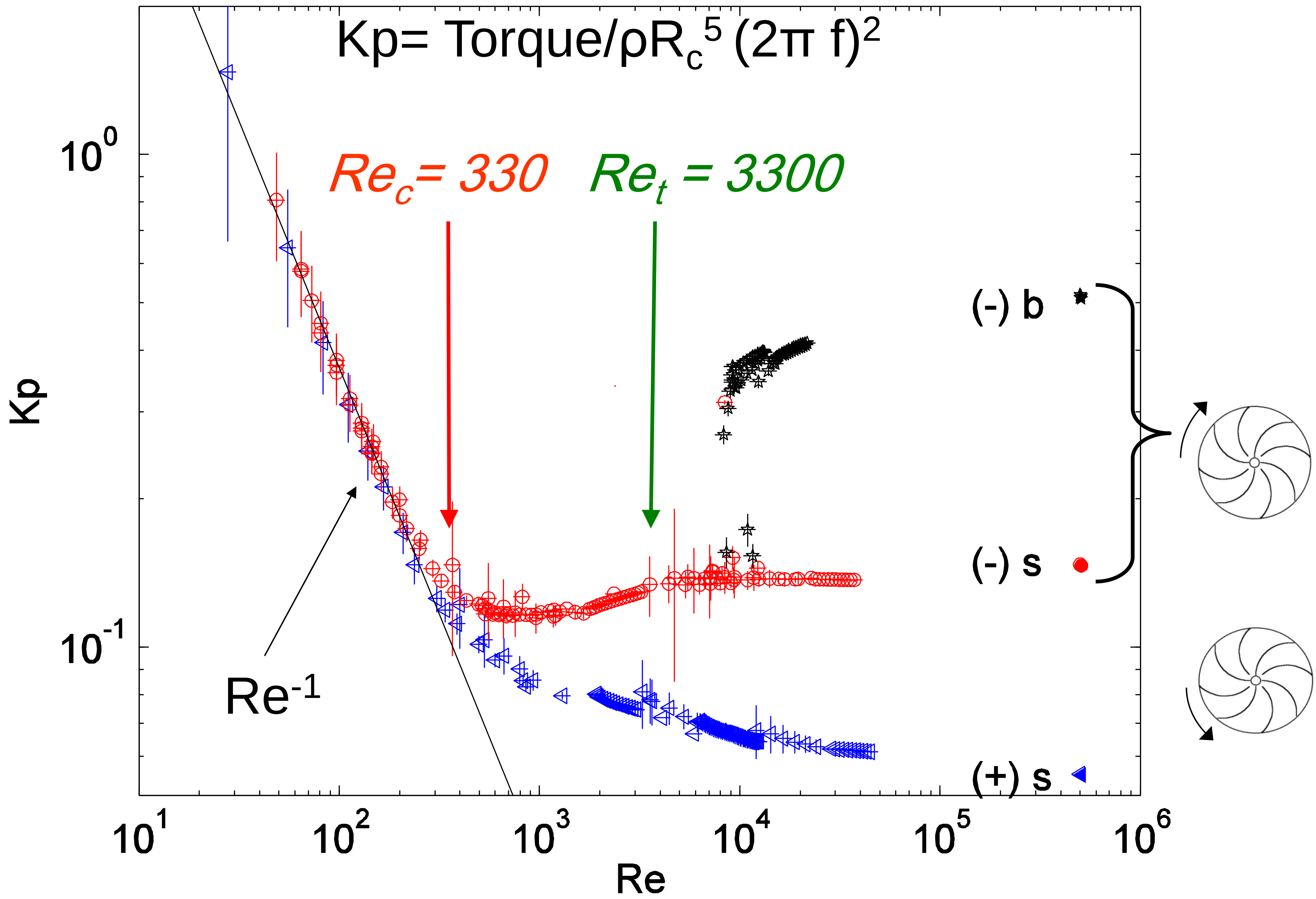


○ : $Re = 800$
▽ : $Re = 5\,000$
★ : $Re = 10\,000$

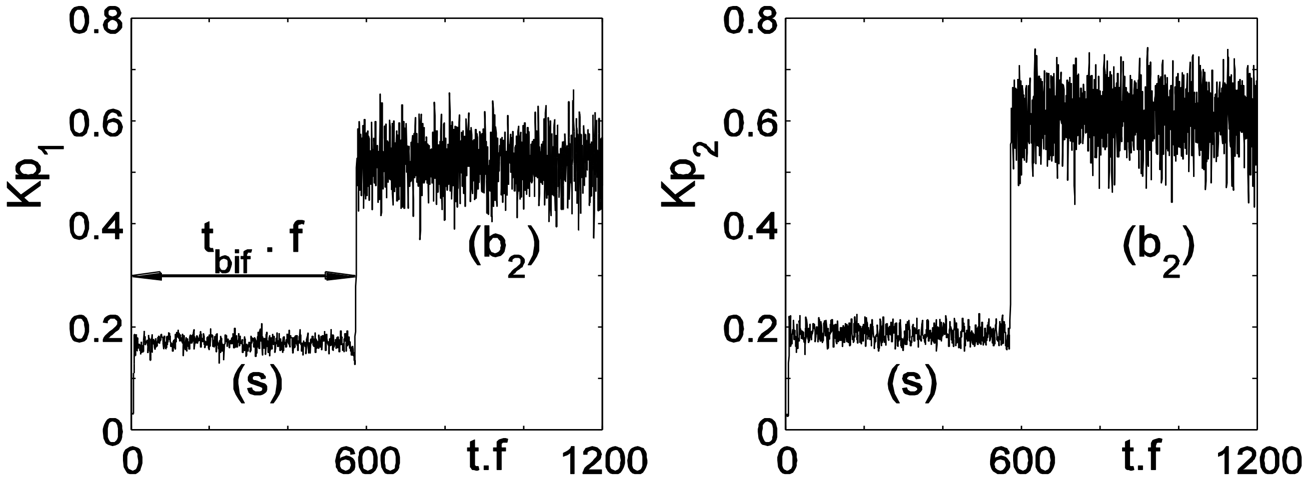


$Re = 3 \times 10^5$

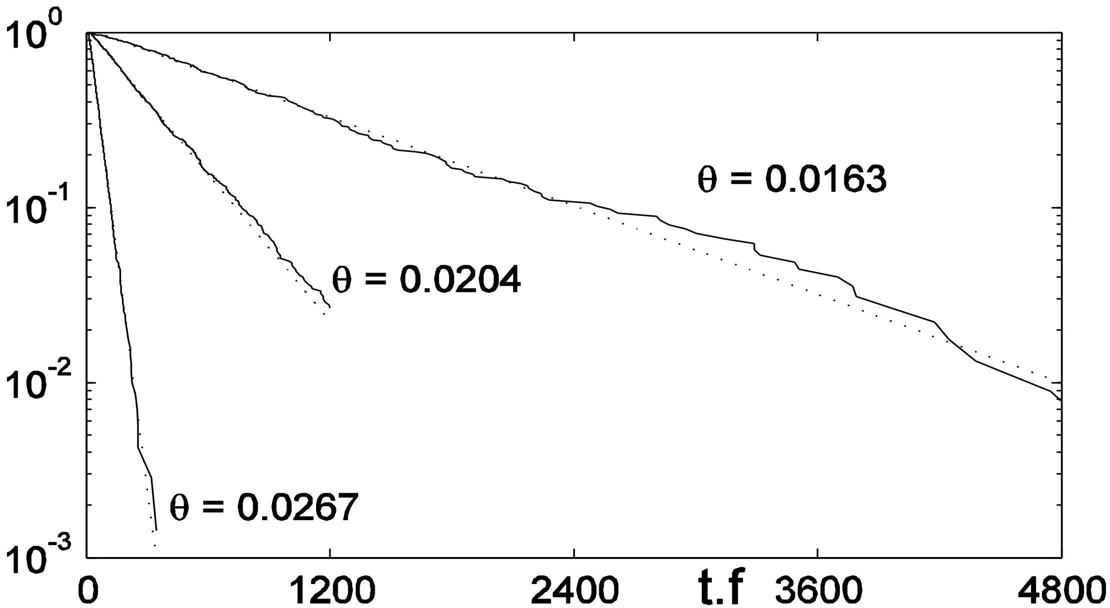
VK flow: multiplicity of solutions



Stability of the symmetric state



Statistics on 500 runs for different θ



Cumulative distribution functions of bifurcation time t_{bif}:

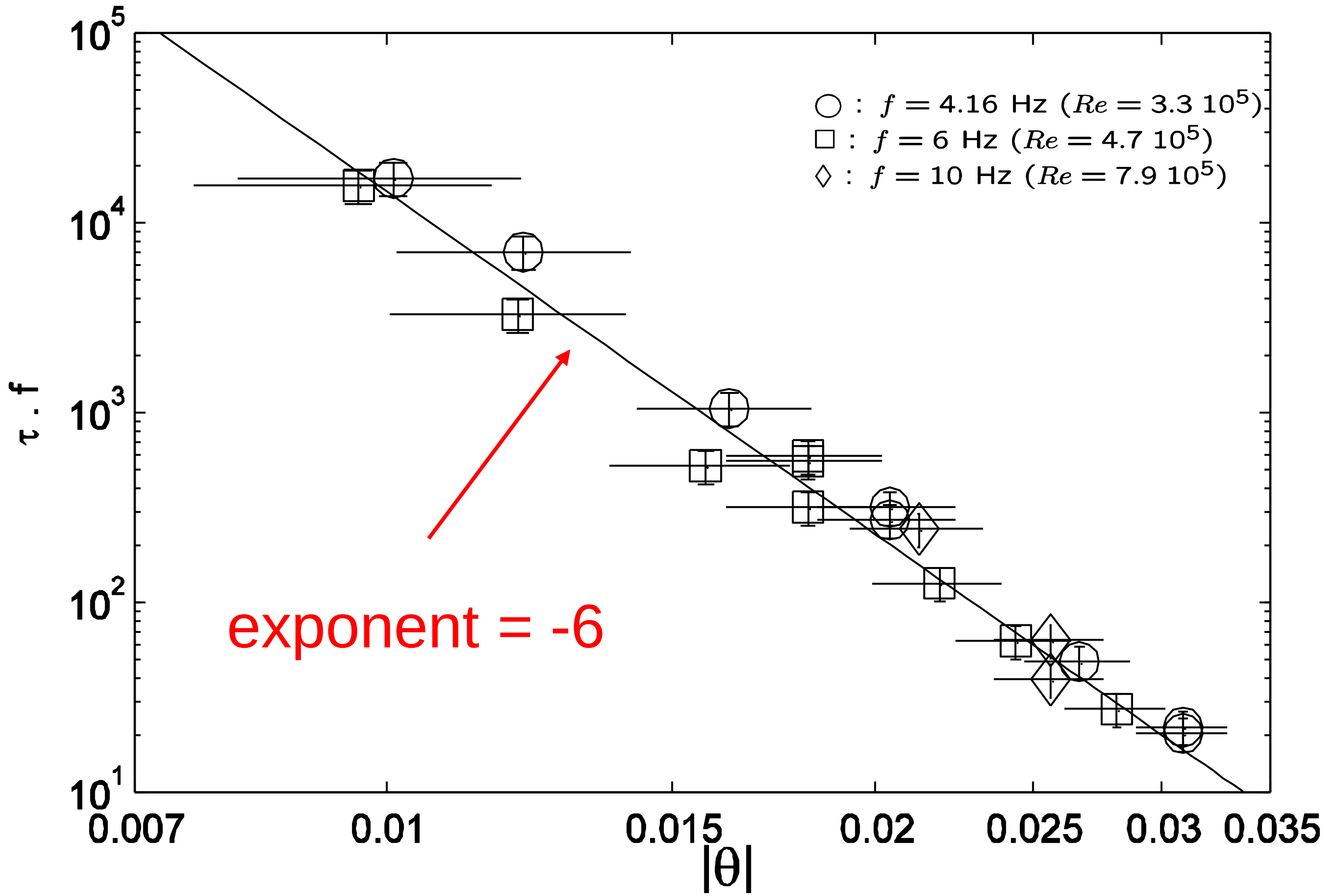
$P(t_{bif}>t)=A\ exp(-(t-t_0)/\tau)$

- $t_0 f \sim 5$
- τ : characteristic bif. time

Stability of the symmetric state

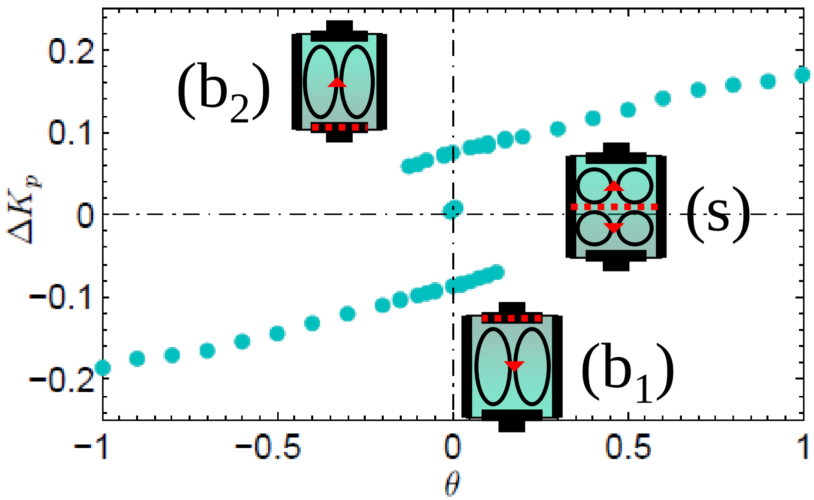
- symmetric state marginally stable

$\tau \rightarrow \infty$ when $\theta \rightarrow 0$

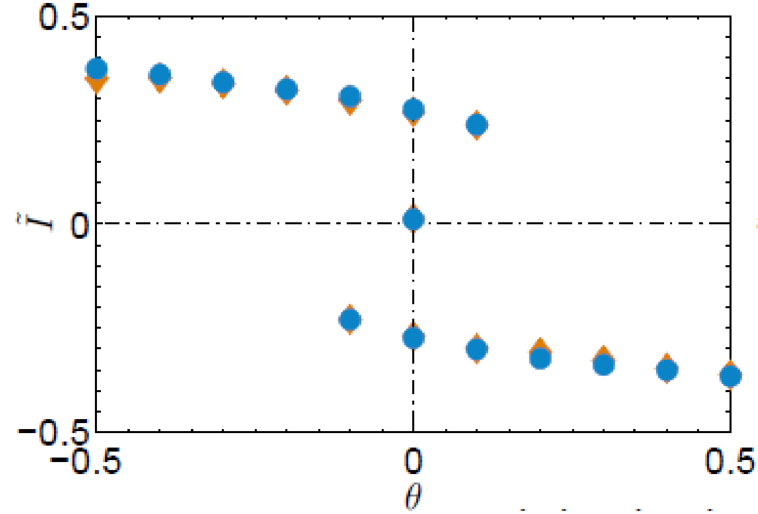


Turbulent bifurcation

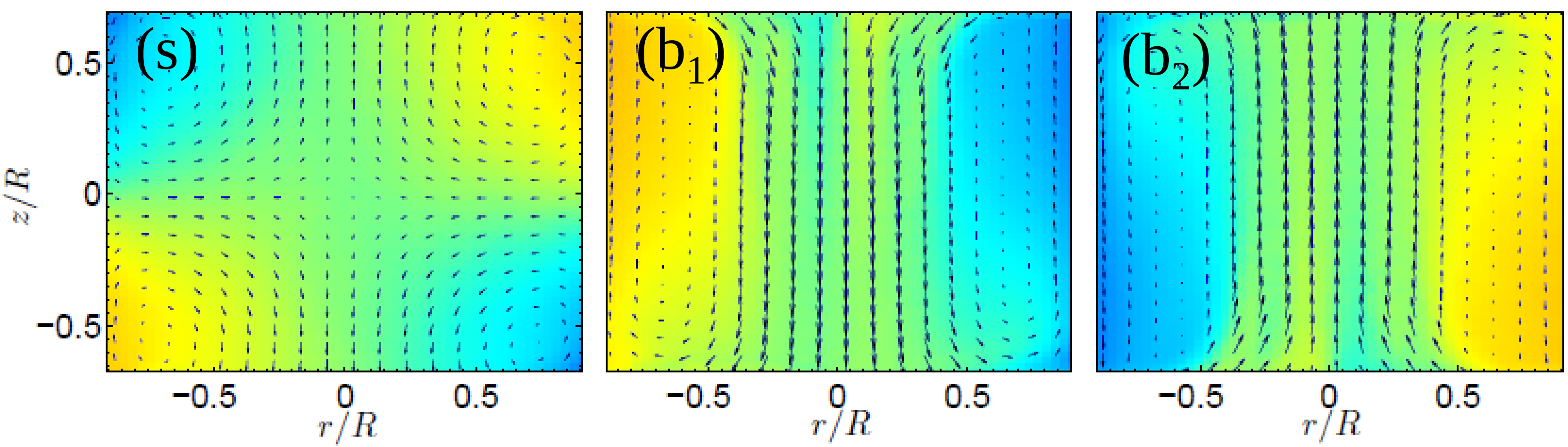
Torque difference



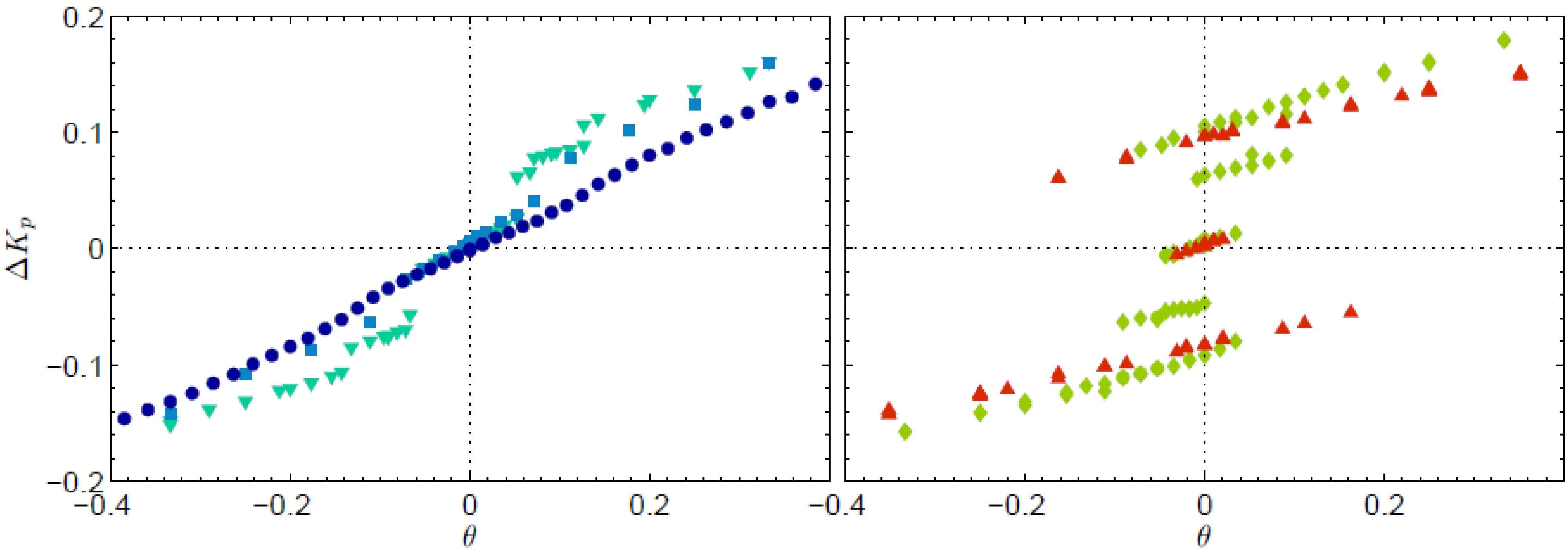
Kinetic momentum



Hysteresis for Re > 16 000

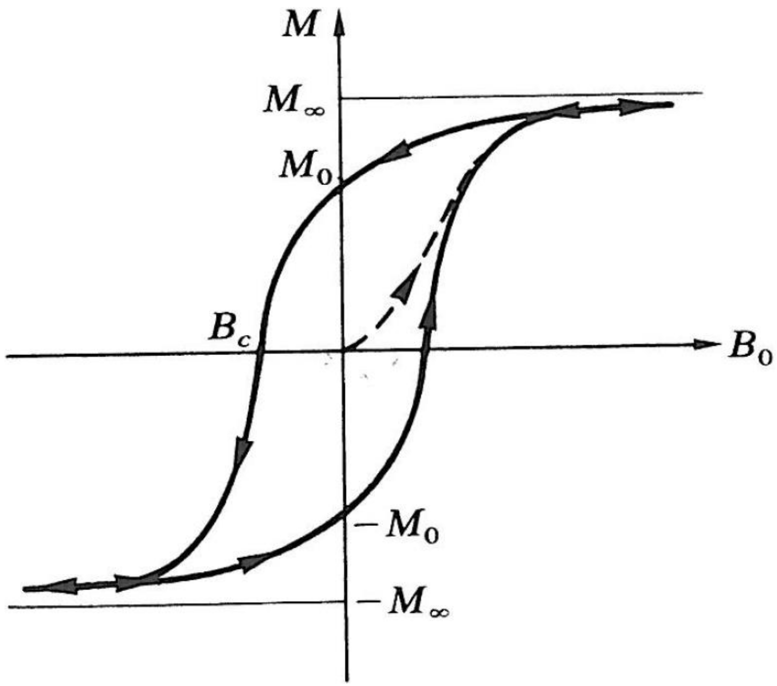


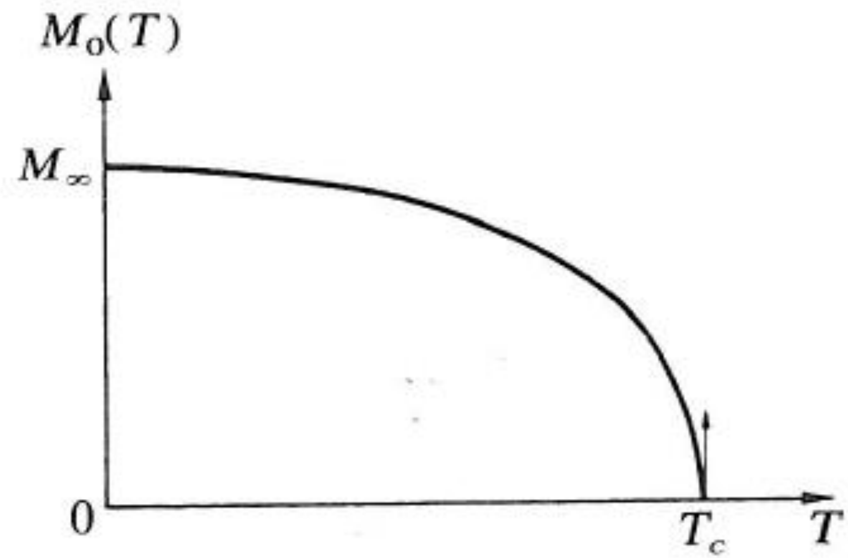
Evolution of ΔKp for different Reynolds



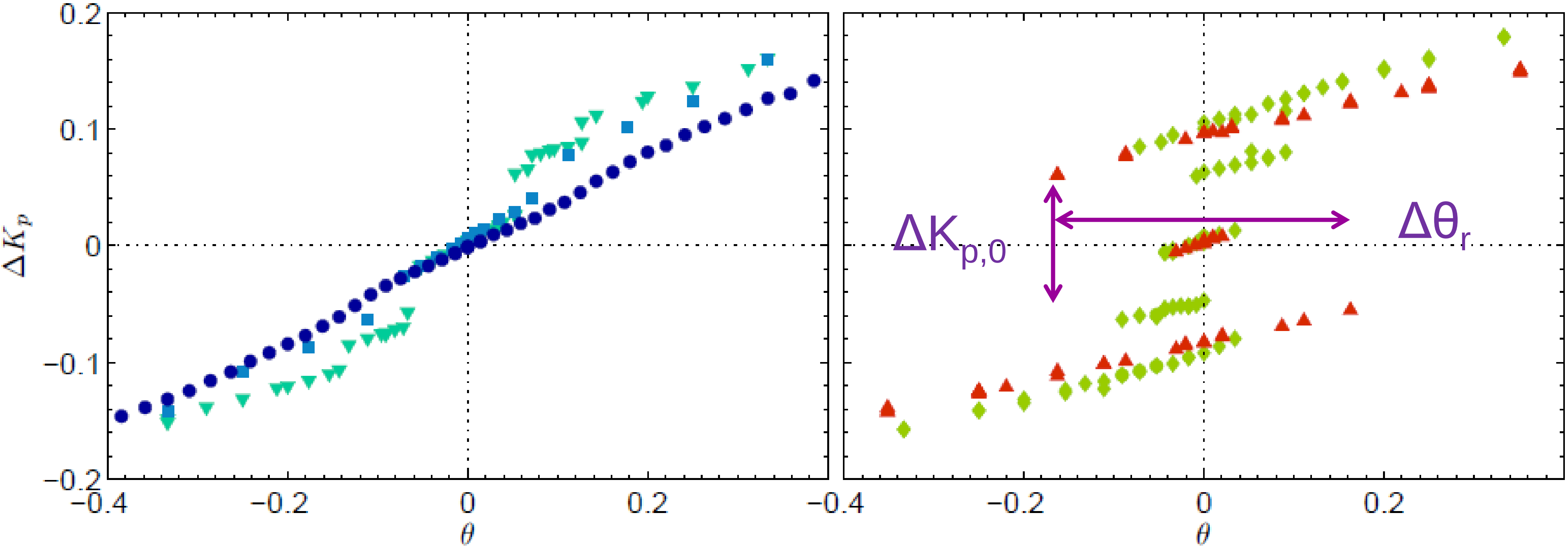
Re= 800 - 3000 – 5800 - 15300 - 195000

Ferromagnetism



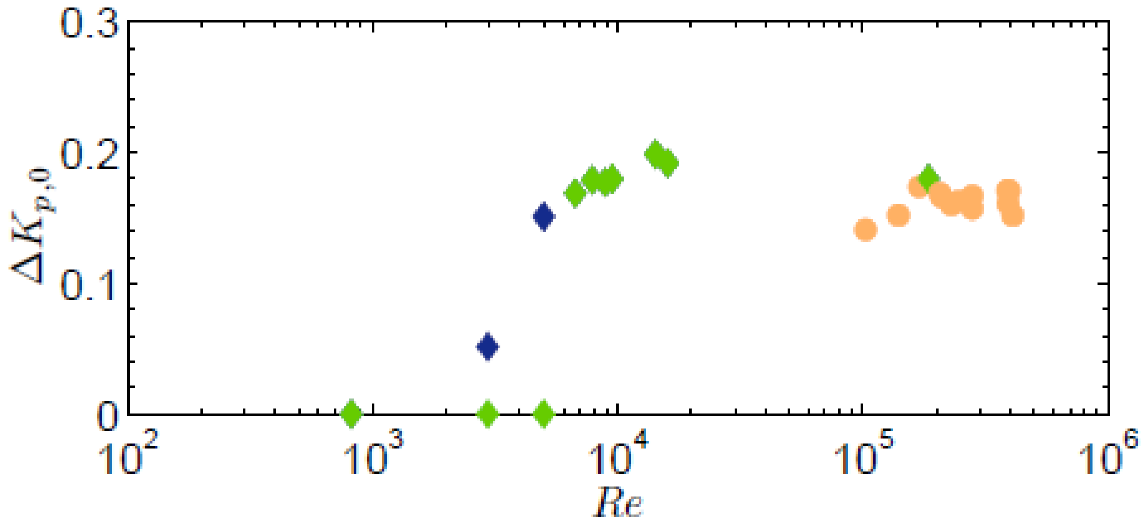


Evolution of ΔKp for different Reynolds

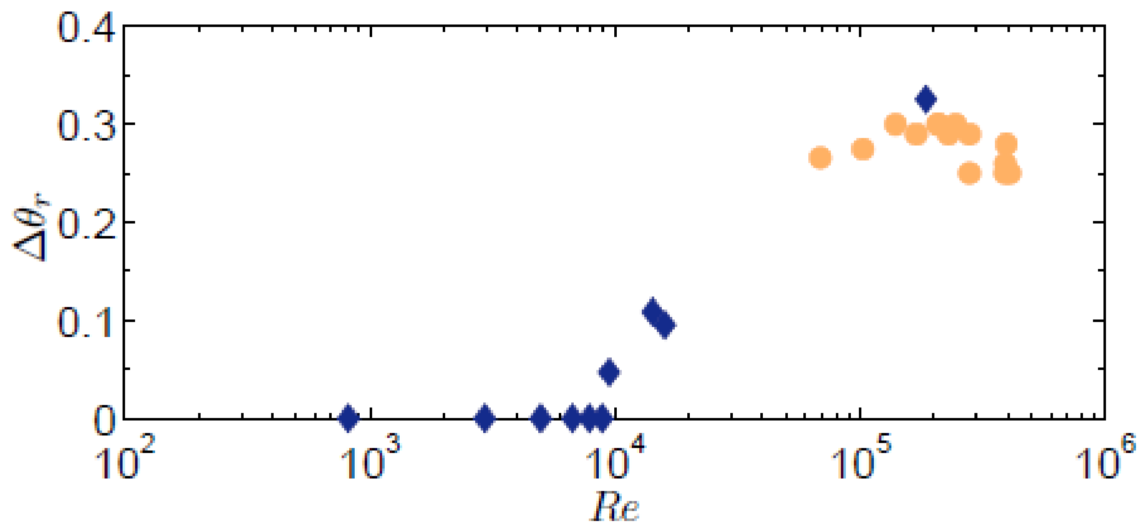


Re= 800 - 3000 – 5800 - 15300 - 195000

Ferroturbulence

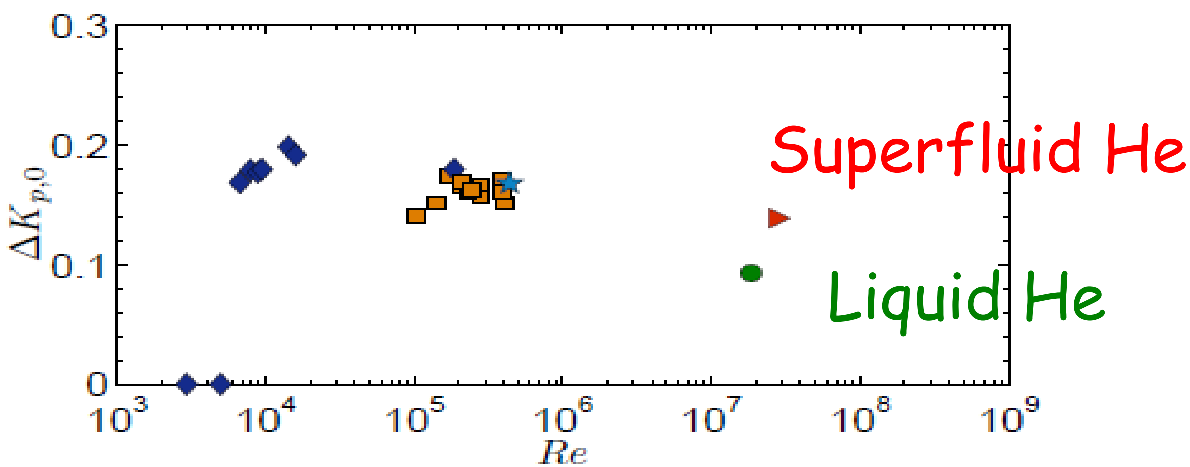


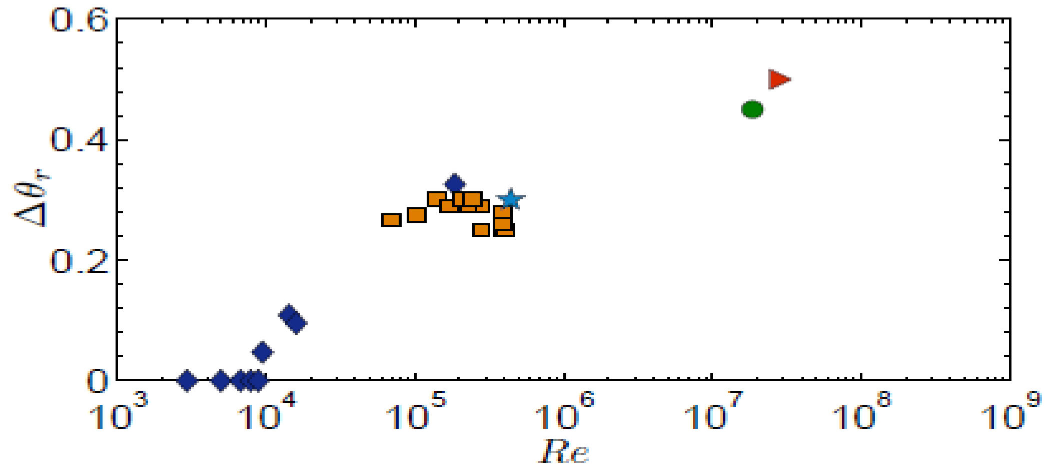
cf. magnetization
M_0 at H=0



cf. coercitive field
B_c at M=0

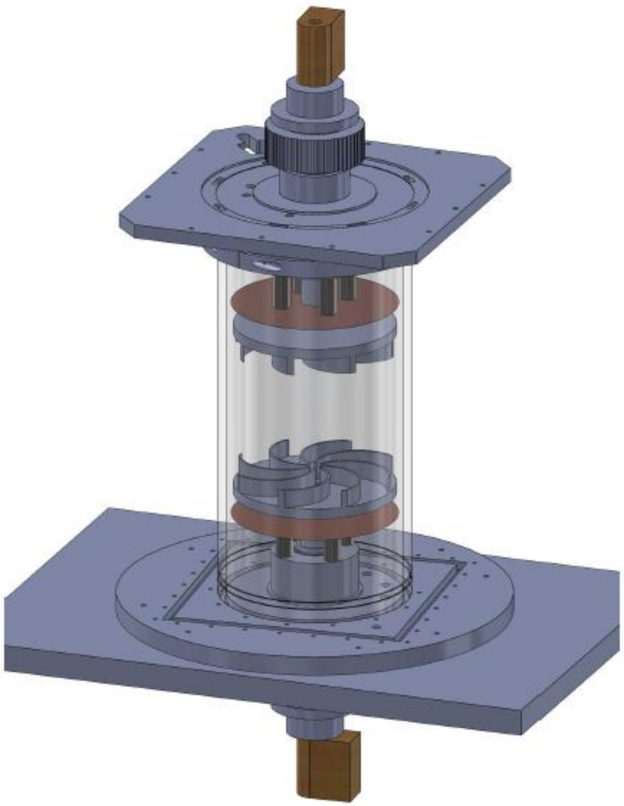
In liquid Helium (SHREK experiment)





Collaboration with ENS Lyon, SBT, Neel, LEGI, LUTh (EuHIT)

Transition to turbulence in von Karman



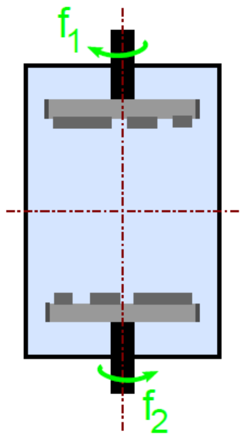
Re

laminar
stationary pattern
oscillating pattern
chaos
turbulence
Eckhaus type instability
Turbulence*
End of story?

Transitions ← θ

forcing

Turbulence and forcing protocols

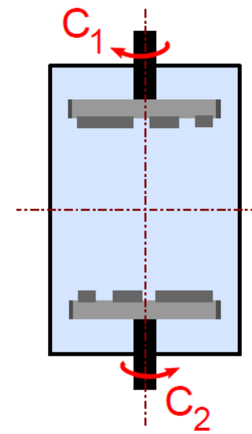


Speed control

Asymmetry control: θ

$$\theta = \frac{f_1 - f_2}{f_1 + f_2}$$

Conjugate parameter: γ



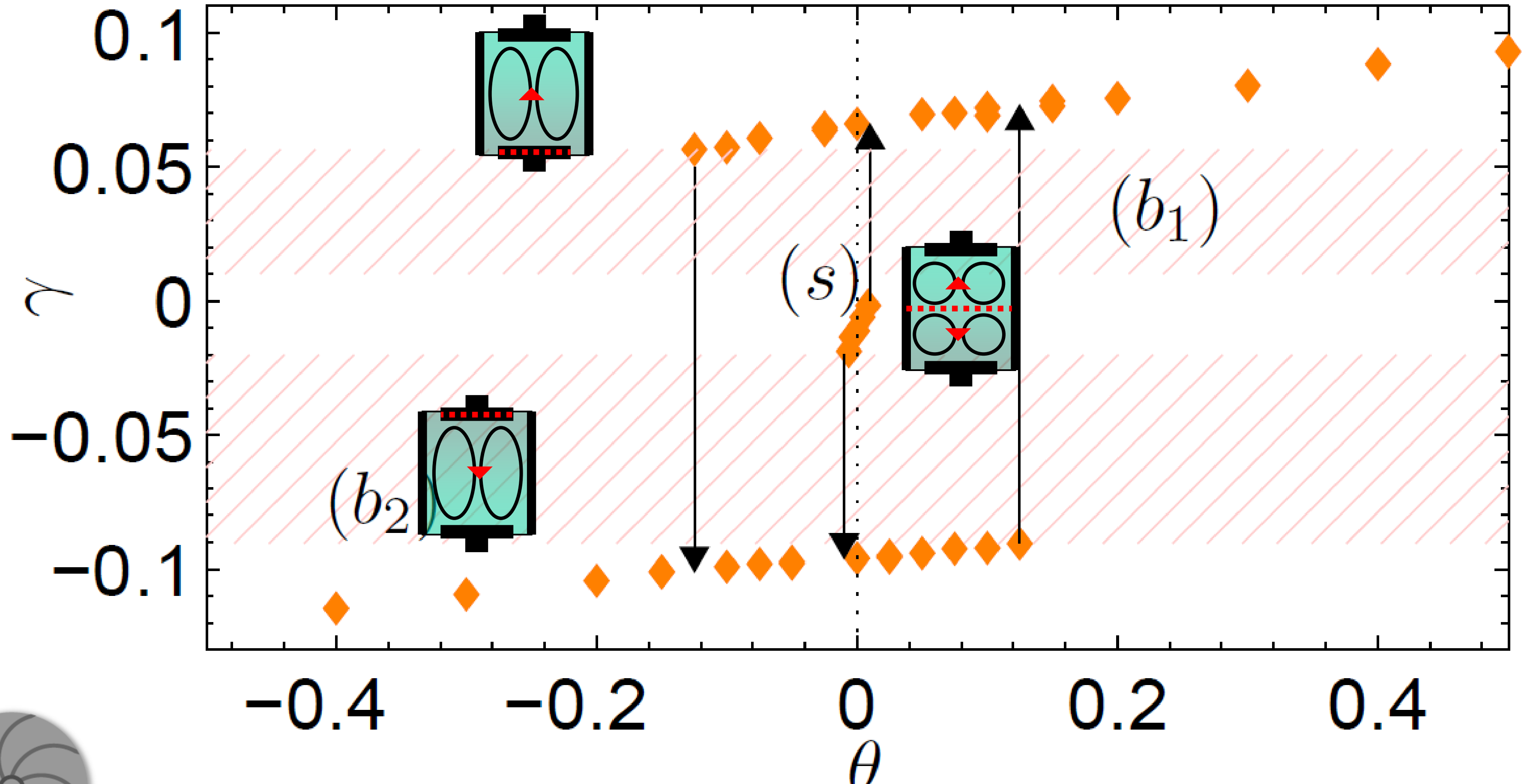
Torque control

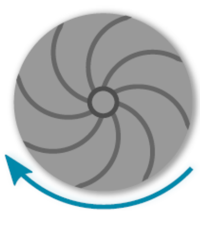
Asymmetry control: γ

$$\gamma = \frac{C_1 - C_2}{C_1 + C_2}$$

Conjugate parameter: θ

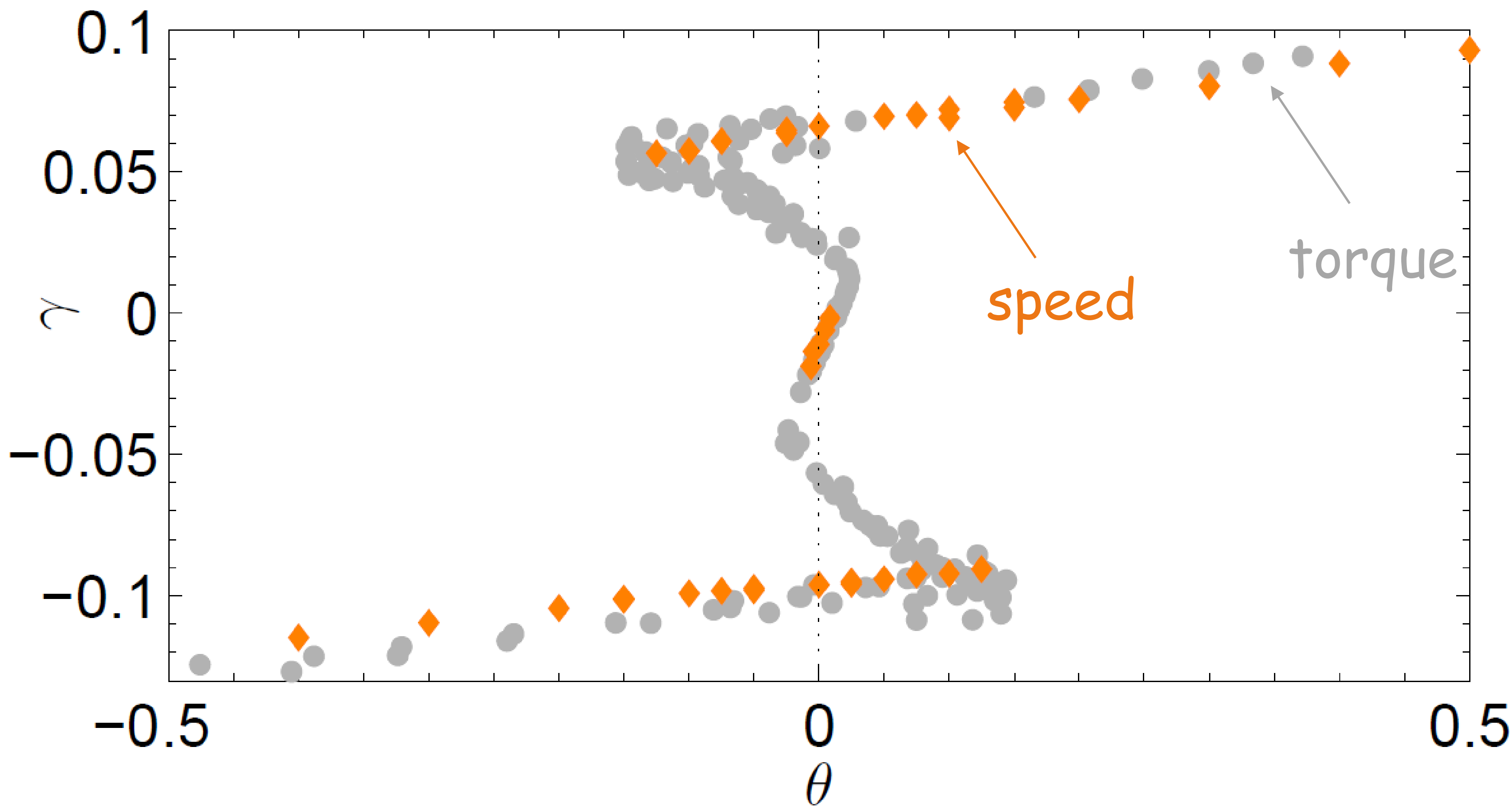
Turbulent bifurcation: speed control





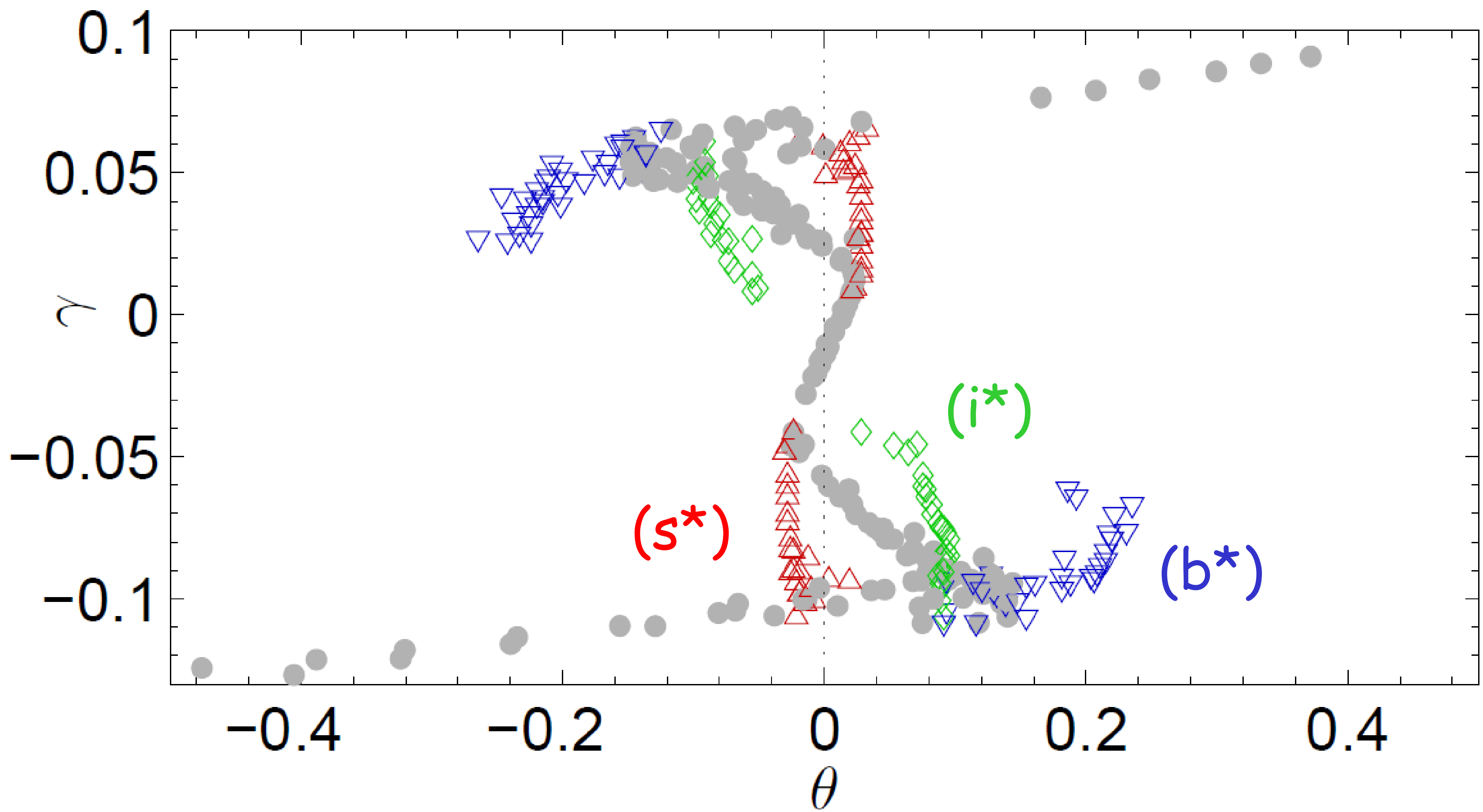
Speed control: forbidden γ zone

Turbulent bifurcation: torque control



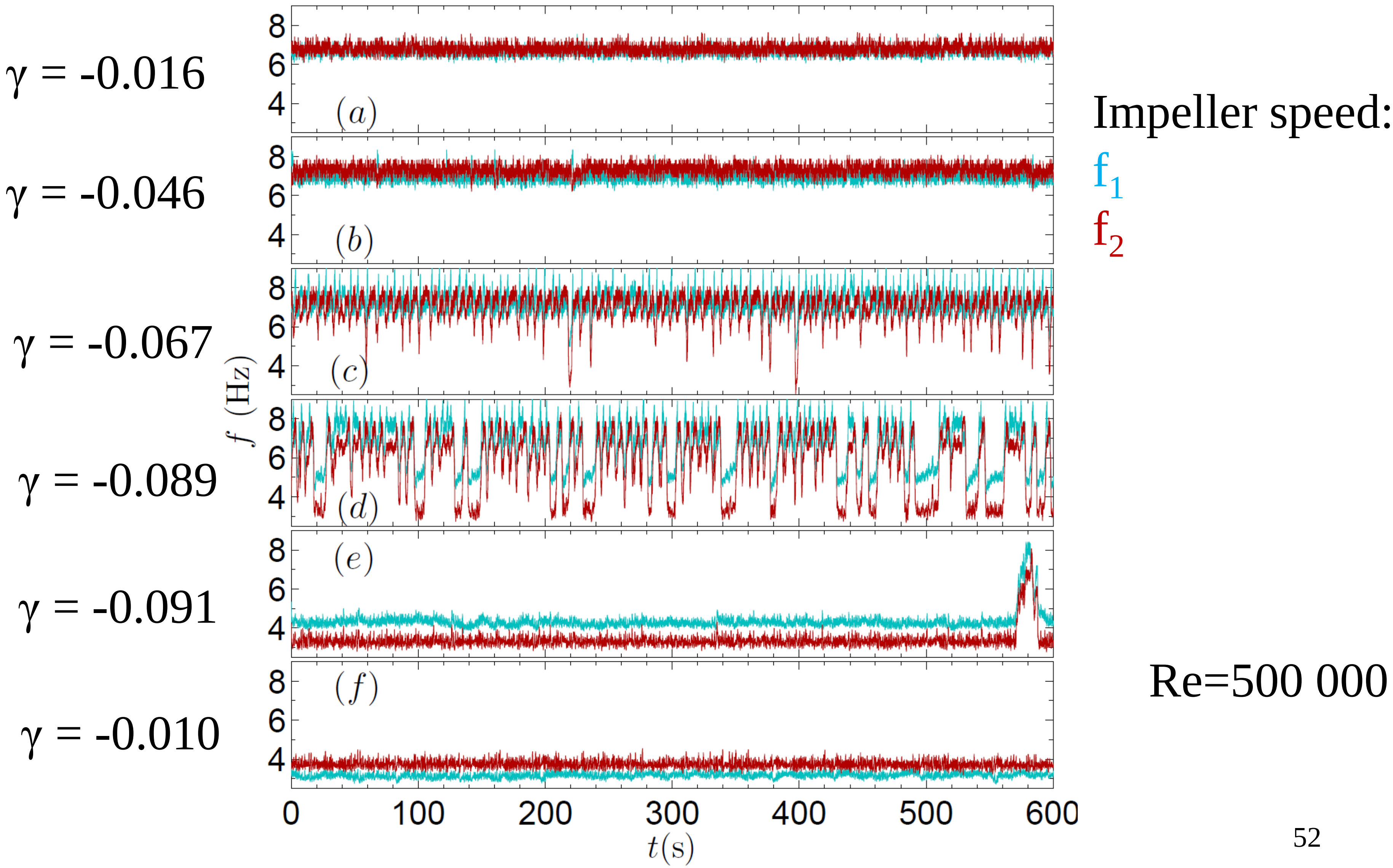
torque control: new mean states accessible

Turbulent bifurcation: torque control

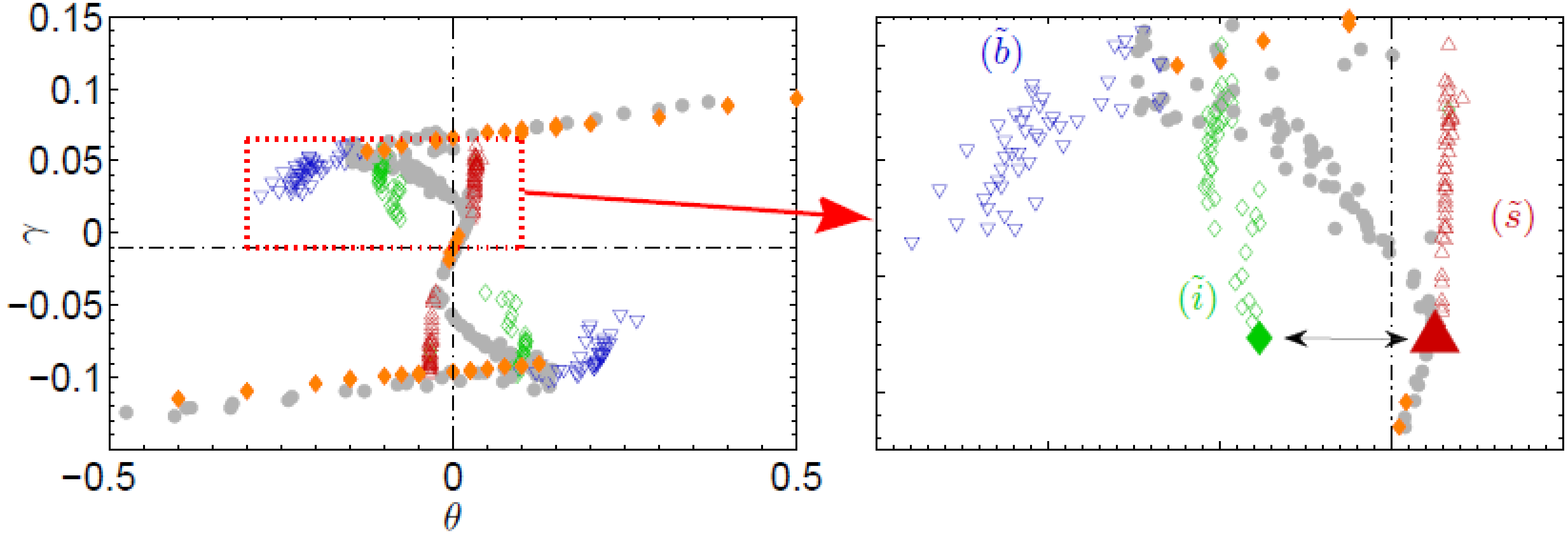


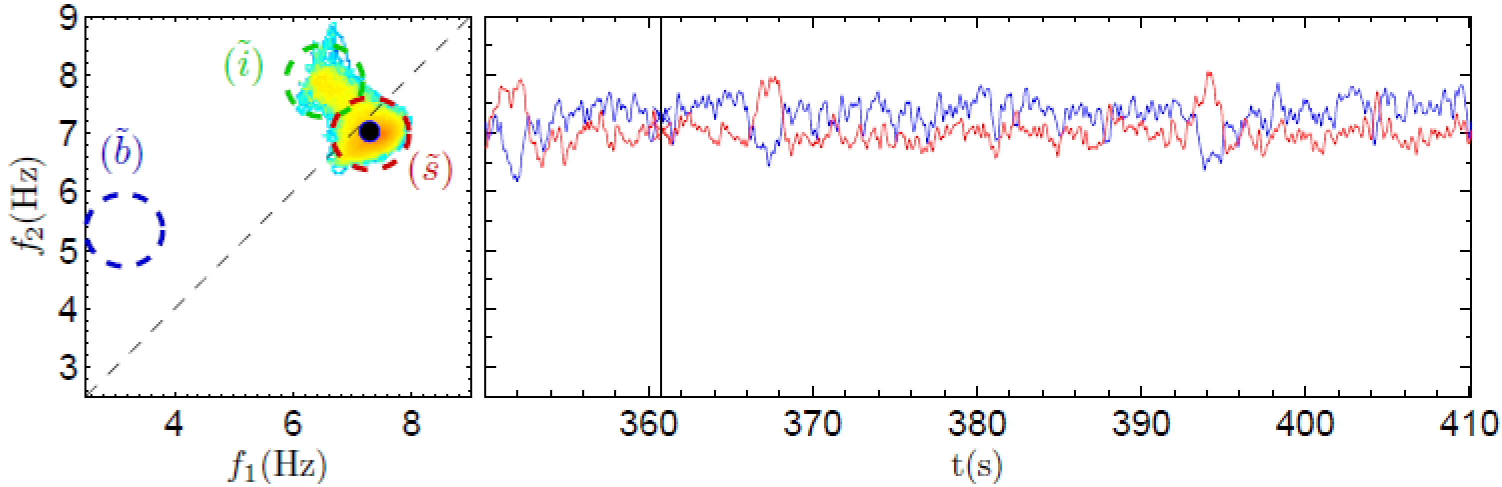
3 states (attractors) (s*) (b*) (i*) in the θ pdf and the joint (f_1,f_2) pdf

Dynamical regimes: torque control

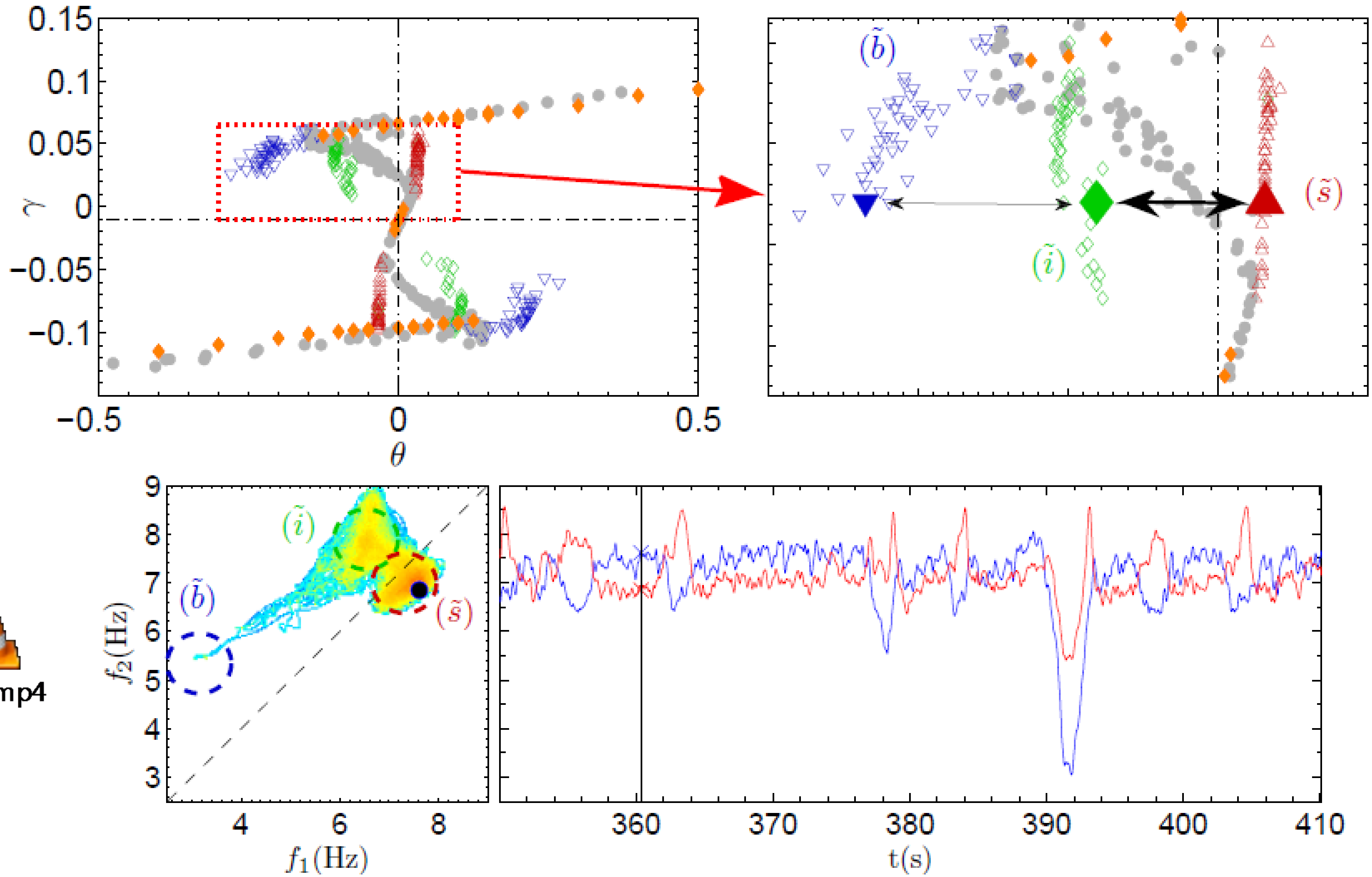


Dynamical regimes: torque control



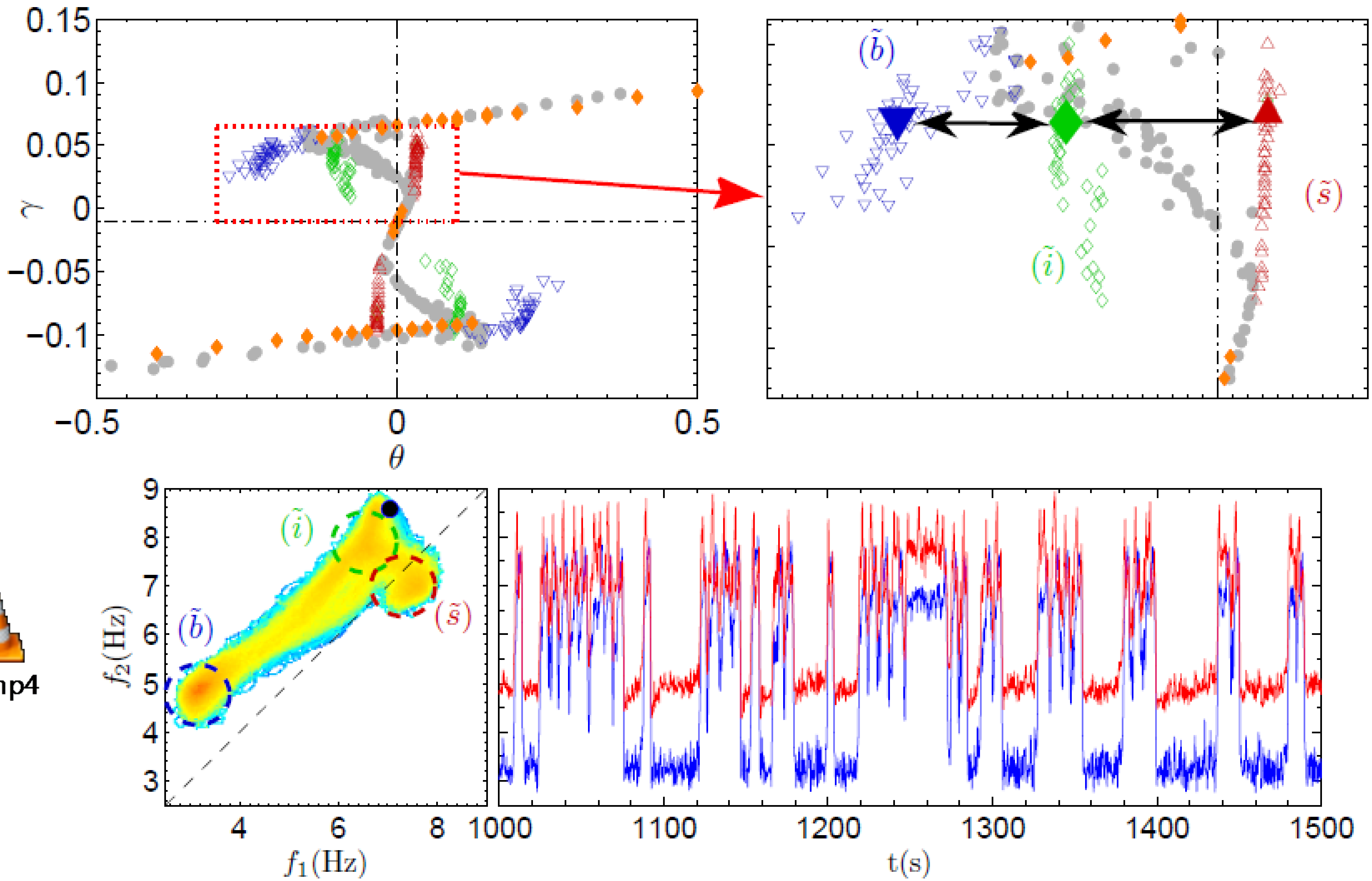


Dynamical regimes: torque control



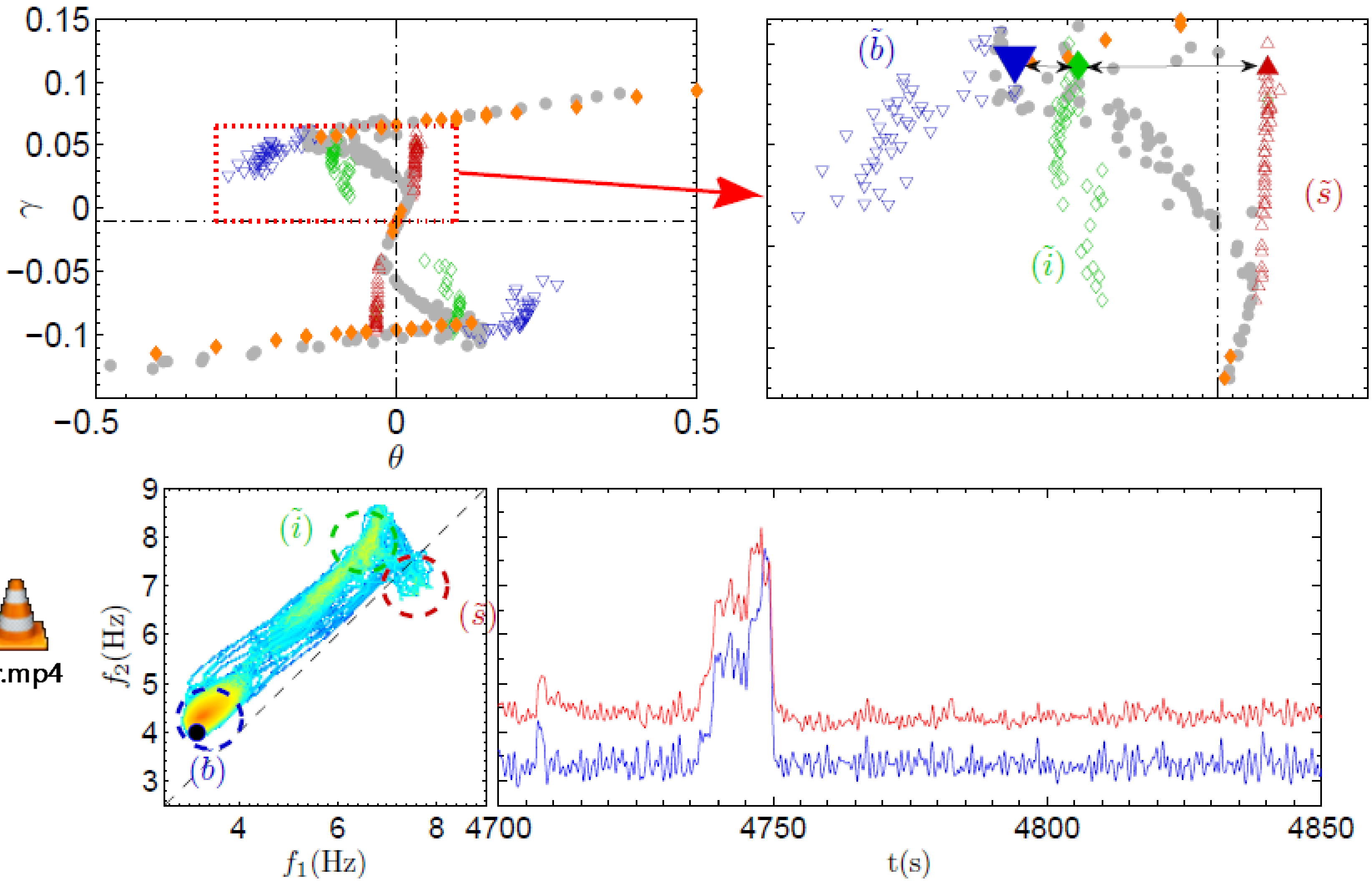
osc2.mp4

Dynamical regimes: torque control



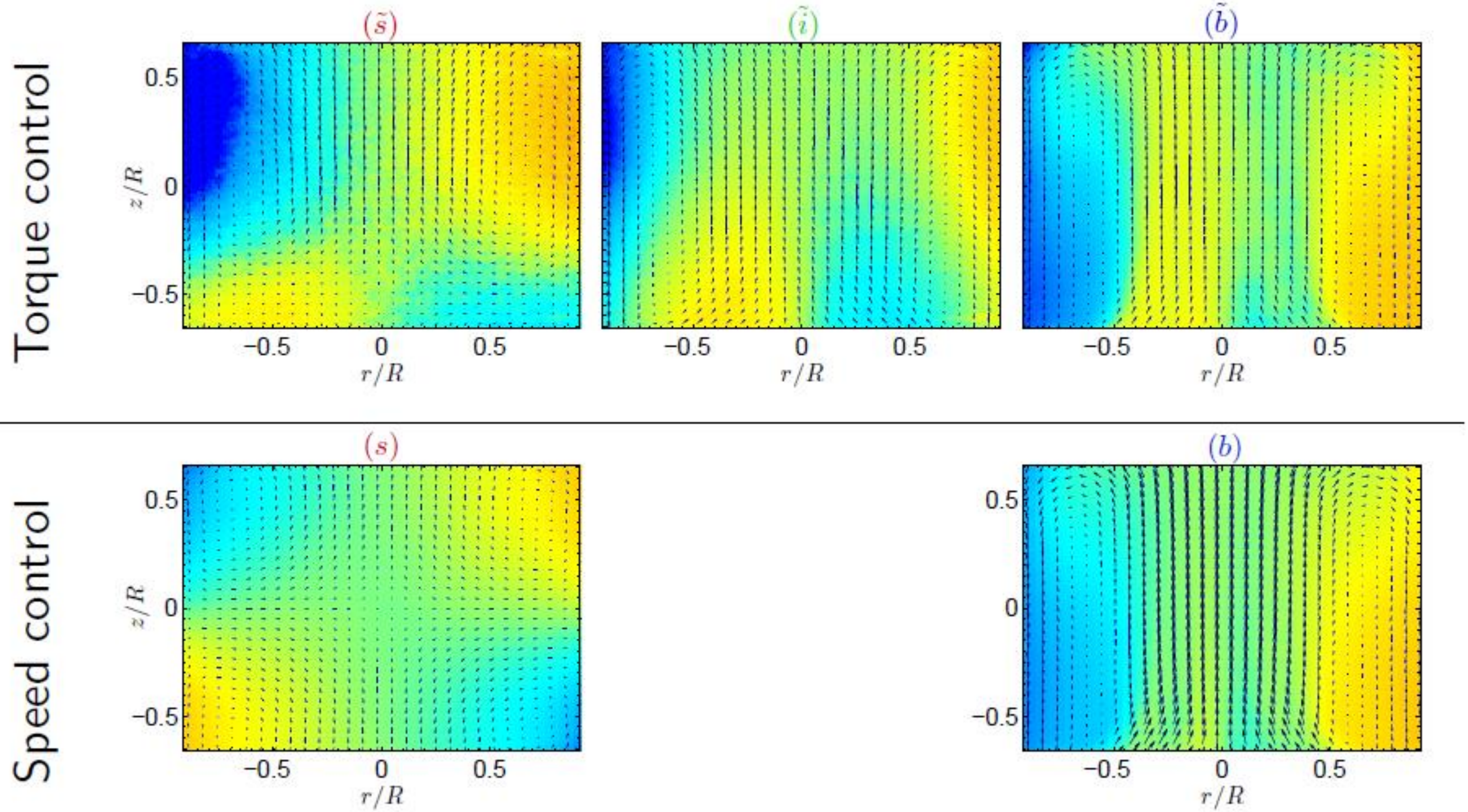
int.mp4

Dynamical regimes: torque control



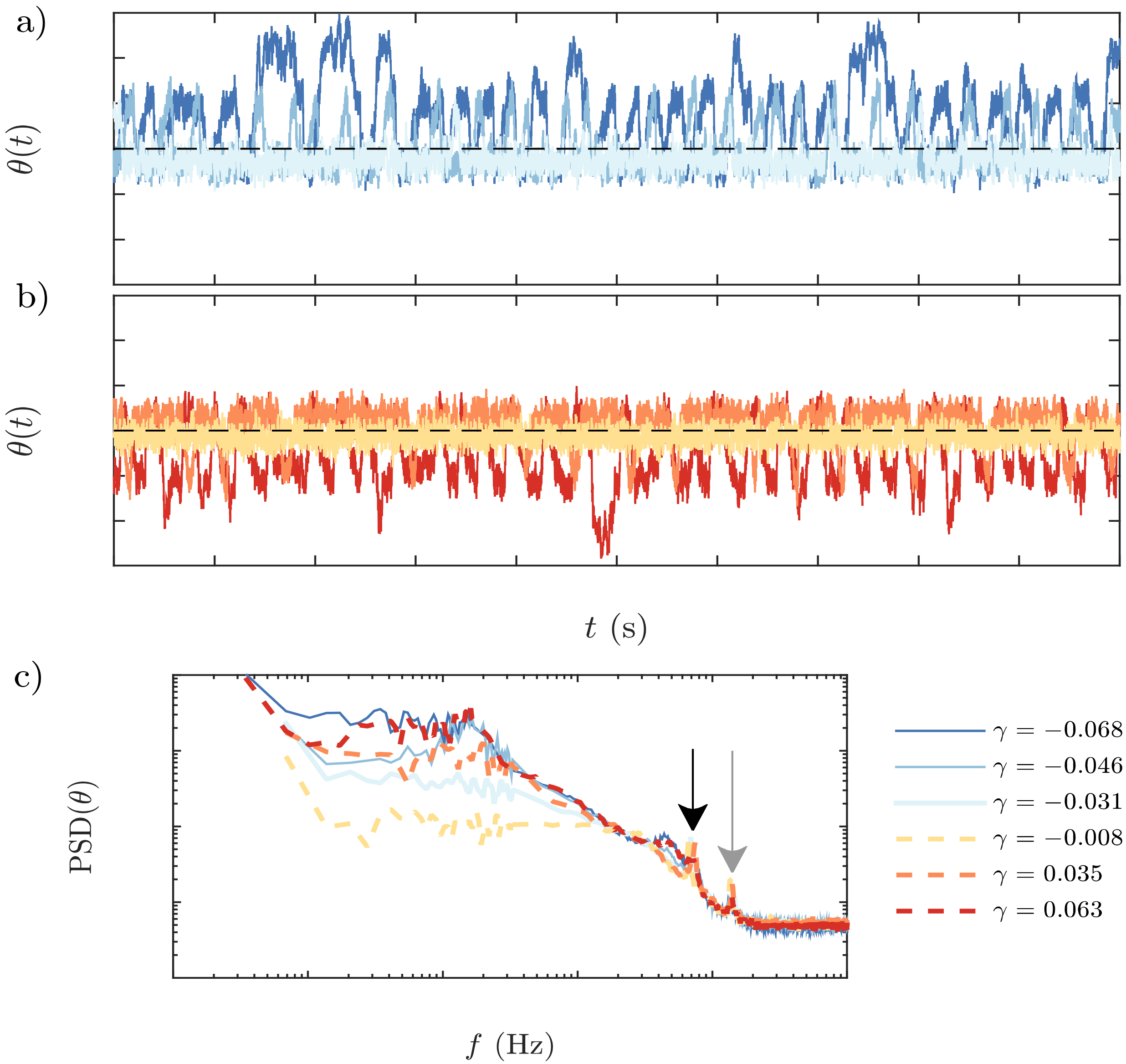
rar.mp4

Torque control: mean velocity fields

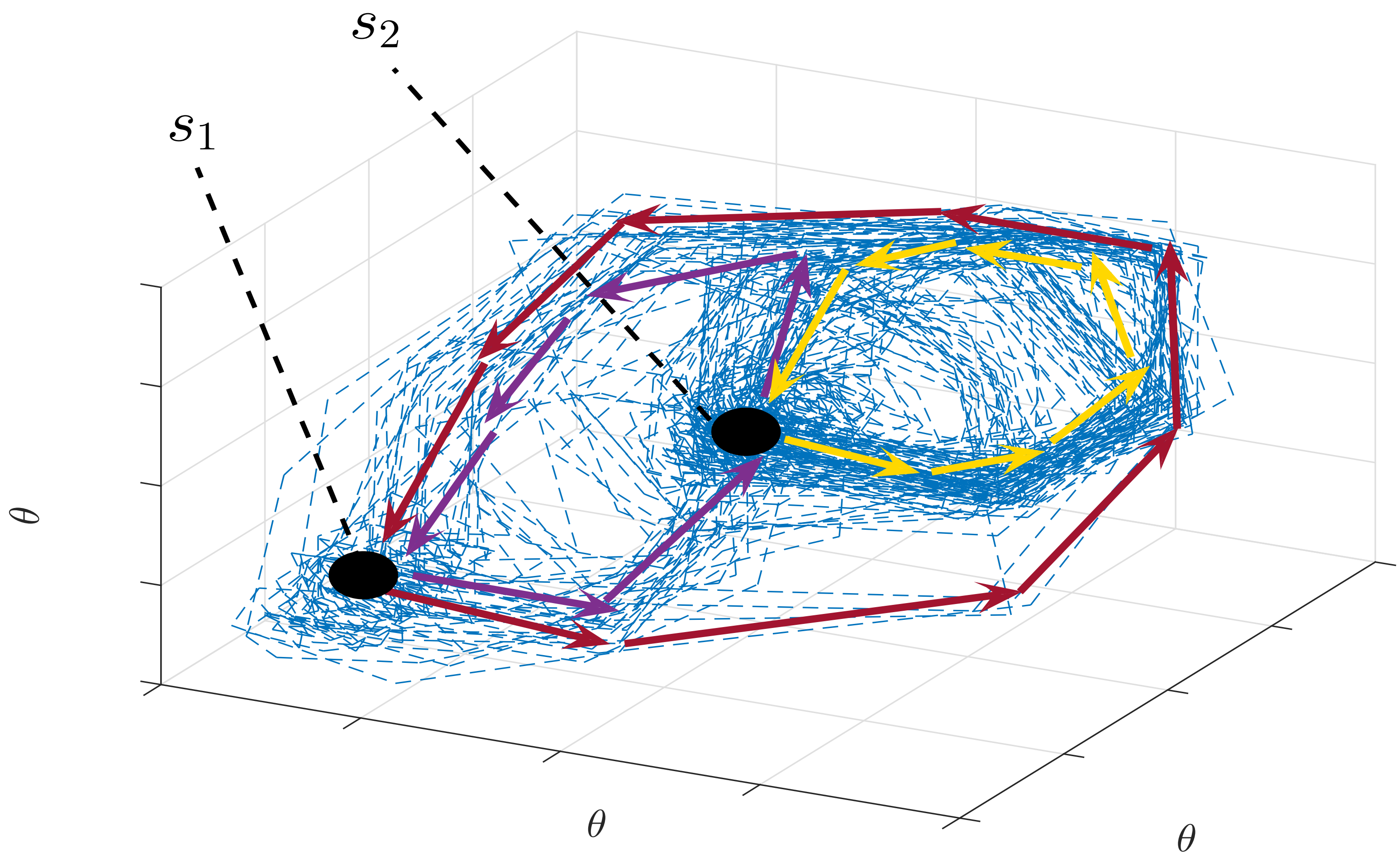


Stability of turbulent states depends on forcing protocol!

Experimental time series



Experimental turbulent attractor



γ=0.067

Torque control: chaos?

Yes but shift of paradigm to stochastic chaos

Classical chaos	Stochastic chaos
Instantaneous velocity	Mean velocity
Bifurcation = Breaking of rotational symmetry	**Bifurcation = Breaking of R_π symmetry of mean flow**
Chaos	Chaos with noise
Attractor	Stochastic Attractor
N=dimension of attractor	N=dimension of attractor+ dimension of noise

Modelisation: stochastic Duffing attractor

minimal dynamical system model *(not reducible to SDE)*

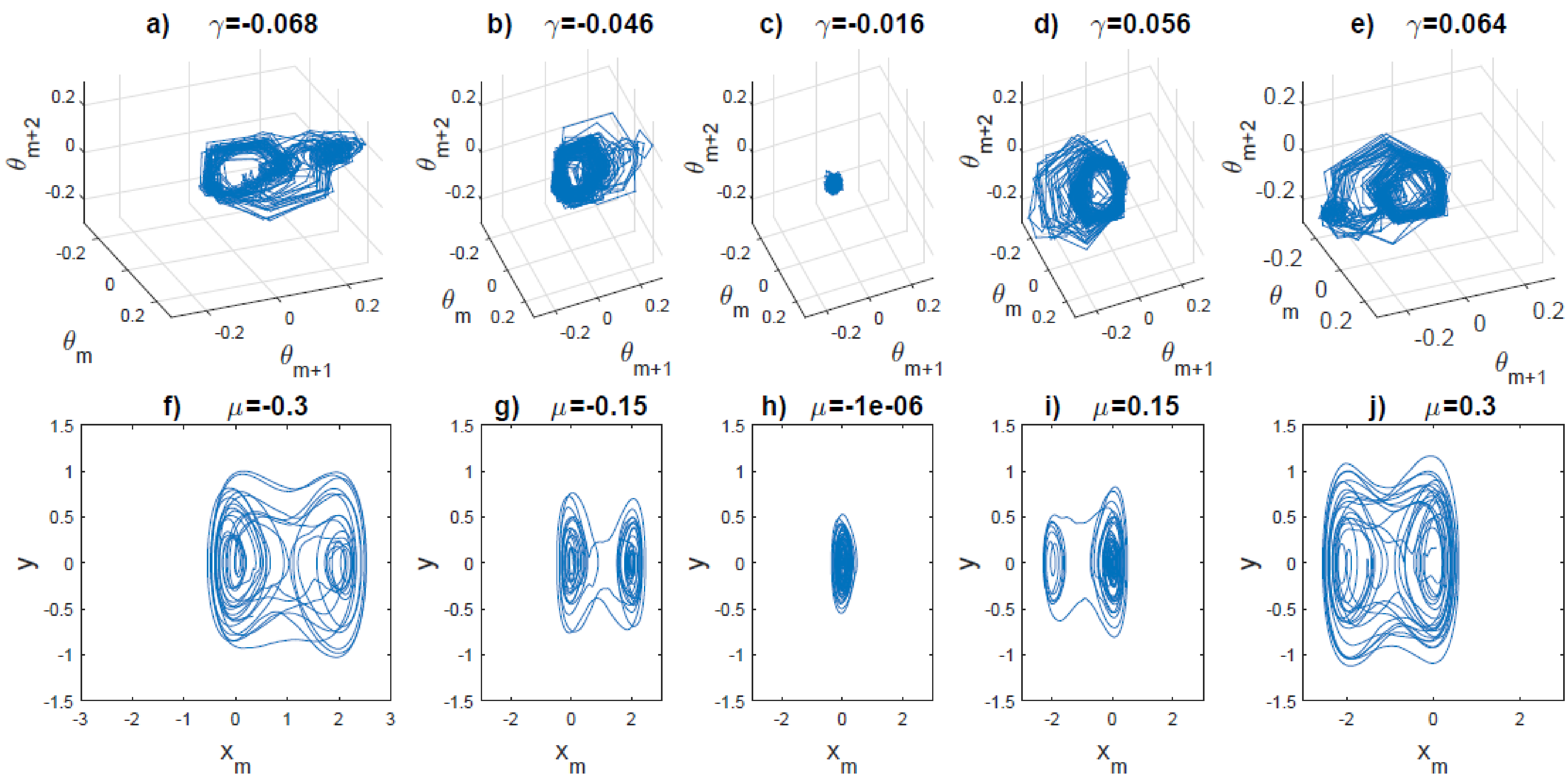
- autonomous oscillator at frequency f_0
- dynamics of γ(t) induced by the turbulent fluctuations represented by a stochastic force.
- θ → - θ symmetry excludes the presence of a quadratic non-linearity

Stochastic Duffing equations, with two variables x (exp: θ) and $y = \dot{x}$, with random forcing z (exp: dynamics of γ(t)) obeying:

$$
\begin{aligned}
dx &= y dt \\
dy &= (-ay + x - x^3 + z \sin \omega t) dt \\
dz &= -\phi(z - \mu) dt + \sigma dW_t,
\end{aligned}
$$

Control parameter: μ (exp: γ)

Comparison Turbulent & Duffing attractor

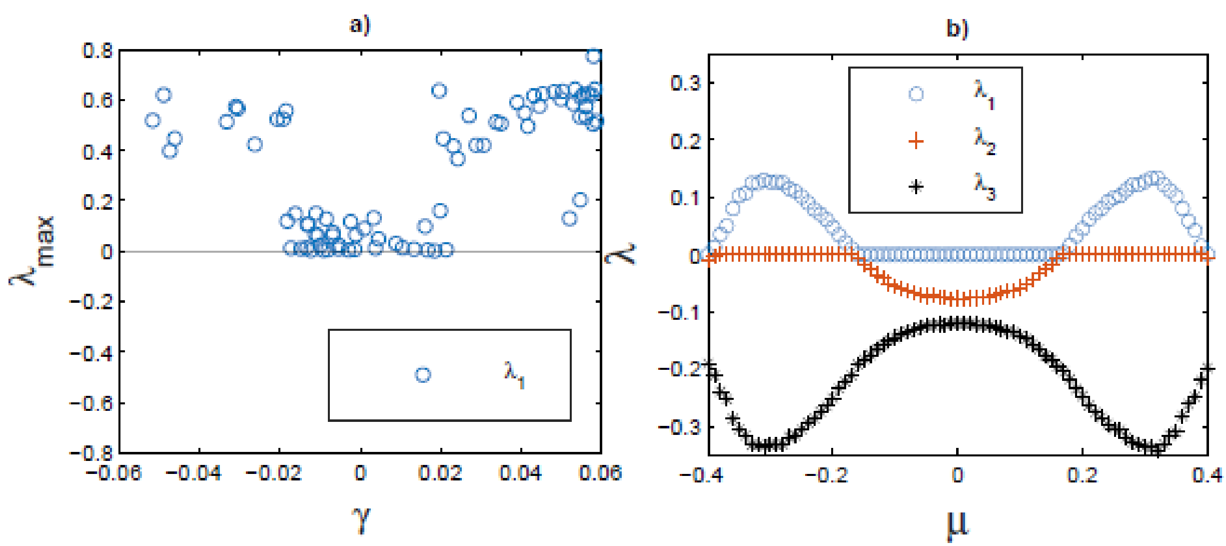


Comparison with Duffing attractor

Effective dimension:

- Turbulent attractor =10
- Duffing attractor = 9

Lyapounov exponents (≠ deterministic Duffing)



Low dimensional turbulence?

→ in turbulent von Karman flows:

- instability of turbulent states f(Re, symmetry,forcing)
- multiplicity of solutions, depending on forcing protocol
- possible transitions between these solutions:
  1st or 2nd order like
- emergence of low dimensional dynamics
- turbulent chaotic attractors


Chaotic turbulence?