Aspects of the Equivalence Principle

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Quantum to Cosmos 6
Nice, October 15 – 17, 2013

Outline

1. The Equivalence Principle

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2. Implications of the UFF

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3. Order of equations of motion

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4. Finsler geometry – Existence of inertial systems

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5. Summary and Outlook

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The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

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Gravity is a metrical theory

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Implication

Gravity is a metrical theory

$\Rightarrow$

Predictions for metrical theories

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense–Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
- Gravitational waves

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General Relativity

Description of tests of the universality principles

Purpose: parametrization of deviations, comparison of different experiments

Haugan formalism (Haugan, AP 1979)

Ansatz: effective atomic Hamiltonian (from modified Dirac and modified Maxwell)

$$H = mc^2 + \frac{1}{2m}\left(\delta^{ij} + \frac{\delta m_{\mathrm{i}}^{ij}}{m}\right) p_i p_j + m\left(\delta_{ij} + \frac{\delta m_{\mathrm{g}ij}}{m}\right) U^{ij}(\boldsymbol{x}) + \ldots$$

- additional anomalous spin terms (CL, CQG 1996, SME)
- additional anomalous charge terms (Dittus, C.L., Selig, GRG 2006)

can calculate (all quantities depend on all anomalous parameters)

- acceleration $\longrightarrow$ WEP tests
- frequency comparison $\longrightarrow$ redshift tests
- spin dynamics

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Consequences of the UFF

Trajectories

- Trajectory of a particle $x = x(p;t)$
  $p =$ particle parameter (e.g. mass, charge, etc)
- UFF $\Rightarrow$ trajectory does not depend on particle parameters $x = x(t)$
  This is already the geometrization of the gravitational interaction
- The set of all trajectories is a path structure

Order of equations of notion / Cauchy problem

- Newton's setup: trajectory determined through
  - initial position $x_0 = x(t_0)$ and
  - initial velocity $v_0 = \dot{x}(t_0)$.

$\Rightarrow$ ordinary differential equations of second order: $\ddot{x}^\mu = H^\mu(p; x, \dot{x})$

Question: Why the fundamental equations of motion are of second order?
Equivalent to questioning Newton's second axiom

Consequences of the UFF

UFF + second order

equation of motion

$$\ddot{x}^\mu = H^\mu(x, \dot{x})$$

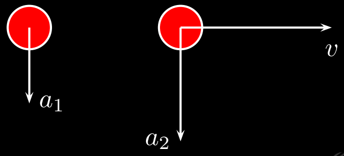
- equation of motion does not depend on particle parameter p
- equation of motion is of second order
- this defines a curve structure

Gravity cannot be transformed away:

Acceleration towards the center of Earth depends on horizontal velocity

exists no inertial system

Implies several effects: $G(T)$, violation of UGR (compare Hohensee, Müller, PRL 2013), ...



The free fall: The notions

Gravity can be transformed away

$\exists$ coordinate system $\forall$ particles : $\ddot{x} = 0$
Then in an arbitrary coordinate system

$$\ddot{x}^\mu = -\Gamma^\mu_{\rho\sigma}(x)\dot{x}^\rho\dot{x}^\sigma$$

autoparallel equation, projective structure (Ehlers, Pirani, Schild 1973, Coleman & Korte, many papers in the 80's)

- Need still relation between the connection $\Gamma^\mu_{\rho\sigma}(x)$ and the metric $g_{\mu\nu}$
  - properties of light and clocks as formulated in EPS axiomatics (Ehlers, Pirani, Schild 1993)
  - free turnability (Helmholtz, Lie)
- result: Riemannian geometry
- How to test whether gravity can be transformed away?
- equivalent to questioning Newton's first axiom

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Order of equation of motion?

Usual framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d}{dt}\boldsymbol{p} = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) \text{ with } \boldsymbol{p} = m\dot{\boldsymbol{x}}$$

more general equations?

- $\boldsymbol{p} = m\dot{\boldsymbol{x}}$ is a constitutive law. Can be more general (as is many cases)

$$\boldsymbol{p} = \boldsymbol{f}(\dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}}, \ldots)$$

  then higher order equations of motion
- Influence of external fluctuations (e.g. space–time fluctuations, gravitational wave background, Göklü, C.L., Camacho & Macias, CQG 2009): generalized Langevin equation with extra force term

$$\int_0^t C(t - t')\dot{x}(t')dt'$$

Order of equation of motion?

Generalized framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d^2}{dt^2}(\epsilon \ddot{\boldsymbol{x}}) = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}})$$

Our specific model

Gauge procedure in order to invent structure of interactions

$$L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \underbrace{-q_0 A_a \dot{x}^a}_{\text{1st order gauge fields}} + \underbrace{q_1 A_{ab} \dot{x}^a \dot{x}^b}_{\text{2nd order gauge fields}}$$

with (Pais–Uhlenbeck oscillator)

$$L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = -\frac{\epsilon}{2}\ddot{\boldsymbol{x}}^2 + \frac{m}{2}\dot{\boldsymbol{x}}^2$$

ϵ additional new particle parameter, dim$\epsilon = \mathrm{kg\,s}^2$

$\epsilon_{\mathrm{QG}} \sim m_{\mathrm{Planck}} t_{\mathrm{Planck}}^2 \sim 10^{-95}\ \mathrm{kg\,s}^2 \qquad \epsilon_{\mathrm{Ce}} \sim m_{\mathrm{Ce}} t_{\mathrm{Ce}}^2 \sim 10^{-71}\ \mathrm{kg\,s}^2$

Equation of motion

simplest case: constant electric field

$$\epsilon \ddddot{\boldsymbol{x}} + m\ddot{\boldsymbol{x}} = q\boldsymbol{E}_0$$

solution in 1D with initial conditions $x(0) = 0$, $\dot{x}(0) = 0$, $\ddot{x}(0) = 0$, and $\dddot{x}(0) = 0$

$$x(t) = \frac{q}{m} E_0 \left(\frac{1}{2} t^2 + \frac{\epsilon}{m} \left(\cos(\omega t) - 1 \right) \right) \qquad \text{small deviation}$$

$$\dot{x}(t) = \frac{q}{m} E_0 \left(t - \sqrt{\frac{\epsilon}{m}} \sin(\omega t) \right) \qquad \text{small deviation}$$

$$\ddot{x}(t) = \frac{q}{m} E_0 \left(1 - \cos(\omega t) \right) \qquad \mathcal{O}(1) \text{ deviation}$$

$$\dddot{x}(t) = \frac{q}{m} E_0 \sqrt{\frac{m}{\epsilon}} \sin(\omega t) \qquad \omega = \sqrt{\frac{m}{\epsilon}} \qquad \text{large deviation}$$

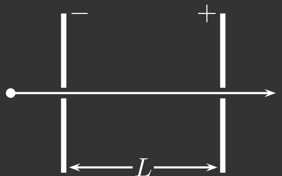
- zitterbewegung
- Limit $\epsilon \to 0$ exists for x and $\dot{x}$, not for $\dddot{x}$

Search for ϵ

Accelerated flight

Flight through accelerator

$$\frac{\langle \dot{x}(L)\rangle - \dot{x}_0}{\dot{x}_0} = \frac{\epsilon}{4m}\frac{\dot{x}_0^2}{L^2}$$



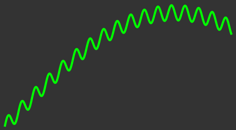
Ion interferometric measurement of acceleration

phase shift

$$\Delta\phi = A(\omega)\boldsymbol{k}\cdot\ddot{\boldsymbol{x}}(\omega)T^2$$

with transfer function

$$A(\omega) = C\frac{\sin^2(\omega t)}{\omega^2}$$

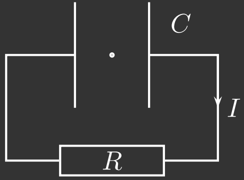


Search for ϵ

Electronic devices

Zitterbewegung of a charged particle induces voltage noise

$$\frac{1}{2} C \langle U^2 \rangle_t = m \langle \dot{x}^2 \rangle = \frac{1}{2} \epsilon \left(\frac{q}{m} E_0 \right)^2$$



- General estimate: $\epsilon \leq 10^{-50}\ \mathrm{kg\,s}^2$.
- Application to mirrors in gw interferometers?
- Adding a small higher derivative term is a mathematical method to analyze differential equations.

C.L. & Rademaker, PRD 2012

higher order time derivative in Schrödinger C.L, Bordé 2000

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Reasons for Finsler geometry

Why Finsler?

- geometry of field equations
- EPS axiomatics (Ehlers, Pirani & Schild 1973)
- dynamical model for respecting UFF but violating Einstein's elevator
- from Quantum Gravity (Girelli, Liberati & Sindoni, PRD 2003)
- VSR (Gibbons, Gomis & Pope, PRD 2007)
- elegance of Lagrange and Hamilton formalism
- nontrivial generalization of Riemannian geometry
- example for violation of Schiff's conjecture
- and Finsler modifications not covered by PPN test theory

Two aspects

- Finsler geometry in the tangent space = Finsler relativity
- Finsler geometry of manifold = Finsler gravity

Finsler geometry

Finsler space

Finsler length function

$$dl^2 = F(x, dx)\,, \qquad F(x, \lambda dx) = \lambda^2 F(x, dx)$$

Finsler metric tensor $f_{\mu\nu}(x, dx)$ is defined as

$$dl^2 = g_{\mu\nu}(x, dx) dx^\mu dx^\nu\,, \quad \text{where} \quad g_{\mu\nu}(x, y) = \frac{1}{2} \frac{\partial^2 F(x^k, y^m)}{\partial y^\mu \partial y^\nu}$$

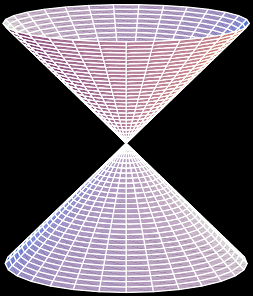
Light cones

Light cone defined by

$$ds^2 = dt^2 - dl^2$$

Finsler geometry

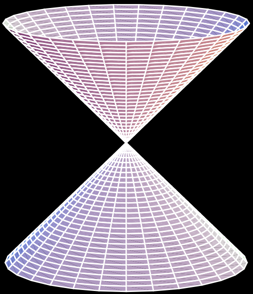
Euclidean light cone | Riemannian light cone | Finslerian light cone



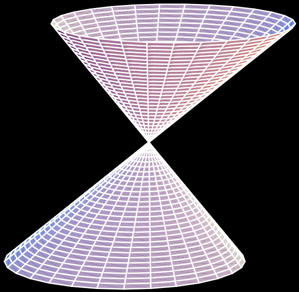
There is no coordinate transformation so that the Finslerian light cone can be locally written in Minkowskian form $0 = -dt^2 + \left(dx^2 + dy^2\right)$

Finsler geometry

Euclidean light cone



Riemannian light cone



Finslerian light cone

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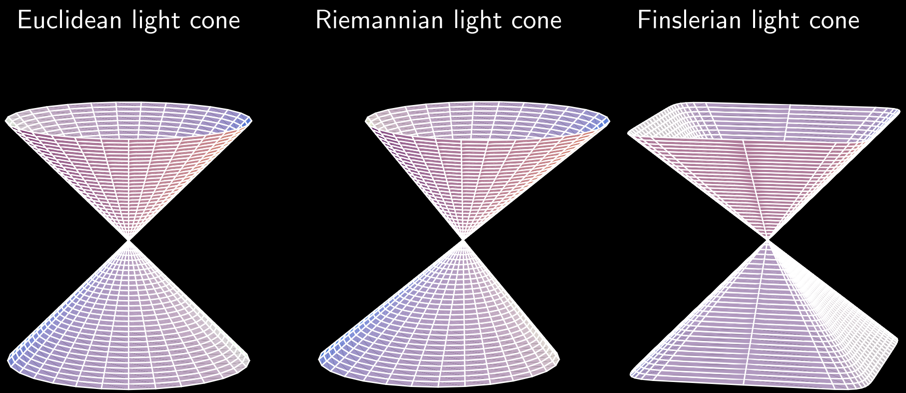
Finsler geometry



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Finsler geometry

Geodesics

$$\delta \int ds = 0 \qquad \Rightarrow \qquad \boxed{0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x}) \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}}$$

with

$$\left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x}) = g^{\mu\nu}(x, \dot{x}) \left(\partial_\rho g_{\sigma\nu}(x, \dot{x}) + \partial_\sigma g_{\rho\nu}(x, \dot{x}) - \partial_\nu g_{\rho\sigma}(x, \dot{x}) \right)$$

- UFF true, but gravity cannot be transformed away (no Einstein elevator)
- violates LLI: counterexample to Schiff's conjecture

Deviation from Riemann geometry

How to describe deviation from Riemannian geometry? (test theory)

Deviation from Riemann (C.L., Lorek & Dittus, GRG 2009)

- Special case: "power law" metrics (Riemann)

$$dl^2 = (g_{\mu_1\mu_2\ldots\mu_{2n}}(x)dx^{\mu_1}dx^{\mu_2}\cdots dx^{\mu_{2n}})^{\frac{1}{r}}$$

- From any given Riemannian metric g_{ij} and a tensor $\phi_{i_1\ldots i_{2r}}$ we can construct a Finslerian metric by

$$\begin{aligned} D^r(dx^i) &= \left(g_{ij}dx^idx^j\right)^r + \phi_{i_1\ldots i_{2r}}dx^i\cdots dx^{i_{2r}} \\ &= \left(g_{i_1i_2}\cdots g_{i_{2r-1}i_{2r}} + \phi_{i_1\cdots i_{2r}}\right)dx^i\cdots dx^{i_{2r}} \end{aligned}$$

- any deviation from Riemann encoded in coefficients $\phi_{i_1\cdots i_{2r}}$
- small deviation given by small $\phi_{i_1\cdots i_{2r}} \ll 1$, then

$$D(dx^i) = g_{ij}dx^idx^j\left(1 + \frac{1}{r}\frac{\phi_{i_1\cdots i_{2r}}dx^i\cdots dx^{i_{2r}}}{(g_{kl}dx^kdx^l)^r}\right)$$

Testing Finsler

1. test of Finslerian Special Relativity:
   - Michelson–Morley type test (C.L., Lorek, Dittus, GRG 2009)
   - quantum tests are under consideration (Itin, C.L., Perlick, in preparation)
2. test of Finslerian gravity: Finslerian deviation from given solutions of Einstein equation

First model: Finsler modification of Schwarzschild

for $h_{\mu\nu}$ Schwarzschild metric: simplest Finsler modification

$$2L = \left(h_{tt} + c^2 \psi_0\right) \dot{t}^2 + \left(\left(h_{ij} h_{kl} + \phi_{ijkl}\right) \dot{x}^i \dot{x}^j \dot{x}^k \dot{x}^l\right)^{\frac{1}{2}}$$

by spherical symmetry

$$\phi_{ijkl} = \psi_1 \dot{r}^4 + \psi_2 r^2 \dot{r}^2 (\sin^2\vartheta \dot{\varphi}^2 + \dot{\vartheta}^2) + \psi_3 r^4 (\sin^2\vartheta \dot{\varphi}^2 + \dot{\vartheta}^2)$$

Solar system: Approximation, Specifications

- linearization with respect to Finslerian perturbations
- restriction to equatorial plane

then

$$L = \frac{1}{2}\left((1+\phi_0)h_{tt}\dot{t}^2 + (1+\phi_1)h_{rr}\dot{r}^2 + r^2\dot{\varphi}^2 + \phi_2 \frac{h_{rr}r^2\dot{r}^2\dot{\varphi}^2}{h_{rr}\dot{r}^2 + r^2\dot{\varphi}^2}\right)$$

with

- $\phi_0 := \frac{c^2}{h_{tt}}\psi_0$ modifies temporal metric
- $\phi_1 := \frac{\psi_1}{2h_{rr}^2}$ modifies radial metric
- $\phi_2 := \frac{h_{rr}\psi_2 - \psi_1}{2h_{rr}^2}$ is “Finslerity” – not covered by standard PPN ansatz

Kepler's third law

for circular orbits

$$\frac{r^3}{T^2}\left(1 - \frac{c^2 r^2}{2GM}\left(\phi_0\left(1 - \frac{2GM}{c^2 r}\right)\right)'\right) = \frac{GM}{4\pi^2}$$

from observations

$$r_1 \left| \frac{\phi_0(r_2) - \phi_0(r_1)}{r_2 - r_1} \right| \leq 10^{-16}$$

for all r_1 and r_2 between Mercury and Neptune

Radial acceleration

acceleration from rest

$$\frac{d^2 r}{d\tau^2} = -\frac{GM}{r^2}\left(1 - \phi_1 - \phi_0' r \left(1 - \frac{c^2 r}{2GM}\right)\right)$$

from observations

$$|\phi_1(r)| \leq 10^{-6}$$

so far no effect related to Finslerity

Effects for Finslerity

- for access to the Finslerity one needs $\dot\varphi \neq 0$ and $\dot r \neq 0$
- this is for light deflection, gravitational time delay, perihelion shift
- calculations are a bit involved ....
  - light deflection
    $$|10^4\,\phi_1 + \phi_2| \leq 50$$
    will be improved by Gaia
  - gravitational time delay
    $$|20\,\phi_1 + \phi_2| \leq 10^{-3}$$
  - perihelion shift
    $$|\phi_2| \leq 10^{-3}$$
- effect most pronounced for perihelion shift (periodic motion)

C.L., Perlick, Hasse: PRD 2012

Quantum mechanics in Finsler space

Finslerian Hamilton operator

$$H = H(p) \qquad \text{with} \qquad H(\lambda p) = \lambda^2 H(p)$$

"Power–law" ansatz (non–local operator)

$$H = \frac{1}{2m} \left(g^{i_1 \dots i_{2r}} \partial_{i_1} \cdots \partial_{i_{2r}} \right)^{\frac{1}{r}}$$

Simplest case: quartic metric

$$H = \frac{1}{2m} \left(g^{ijkl} \partial_i \partial_j \partial_k \partial_l \right)^{\frac{1}{2}}$$

Deviation from standard case

$$\begin{aligned} H &= -\frac{1}{2m} \left(\Delta^2 + \phi^{ijkl} \partial_i \partial_j \partial_k \partial_l \right)^{\frac{1}{2}} \\ &= -\frac{1}{2m} \Delta \sqrt{1 + \frac{\phi^{ijkl} \partial_i \partial_j \partial_k \partial_l}{\Delta^2}} \end{aligned}$$

Quantum mechanics in Finsler space

$$H = -\frac{1}{2m}\Delta\left(1 + \frac{1}{2}\frac{\phi^{ijkl}\partial_i\partial_j\partial_k\partial_l}{\Delta^2}\right)$$

- Hughes–Drever: $H_{\text{tot}} = H + \boldsymbol{\sigma}\cdot\boldsymbol{B}$
- Atomic interferometry, atom–photon interaction

$$\delta\phi \sim H(p+k) - H(p) = \frac{k^2}{2m} + \frac{1}{m}\left(\delta^{il} + \frac{\phi^{ijkl}p_j p_k}{p^2}\right)p_i k_l$$

  modified Doppler term: gives different Doppler term while rotating the whole apparatus (even in Finsler light still propagates on straight lines, anisotropy – deformed mass shell)

- incorporation of gravity needs relativistic framework

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Summary

- discussion of underlying assumptions influencing the meaning of UFF and EEP
- order of equation of motion
- Finsler geometry as example for no inertial system / violation of local Minkowski
- no test theory so far for Finslerian modification of gravity, needs considerations beyond PPN
- Finslerian modification of Schwarzschild
- Solar system effects
- Finsler is further example for violation of Schiff's conjecture

Outlook

- Earth–Moon system in field of Sun, should lead to extra polarization, comparison with LLR data
- Finslerian extension of Kerr
- Klein–Gordon in Finsler in order to discuss coupling of Finsler gravity to quantum mechanics $F(\partial)\varphi + m^2\varphi = 0$
- Maxwell equations in Finsler geometry $H^\nu(\partial)F_{\mu\nu} = j_\mu$ (C.L., Perlick, Hasse, PRD 2012)
- Hydrogen atom in Finsler geometry (Itin, C.L., Perlick, in preparation)

Summary

Thank you!

Thanks to

- H. Dittus
- E. Göklü
- D. Lorek
- H. Müller
- V. Perlick
- P. Rademaker

- DLR
- DFG
- Research Training Group "Models of Gravity"
- Center of Excellence QUEST